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MIMO VARIABLE STRUCTURE CONTROL OF A WASTEWATER TREATMENT PROCESS

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Abstract: In this paper, we consider the control problem of a nonlinear system which is considered as a denitrification process used for the biological treatment of wastewater. The bioreactor to be controlled is a nonlinear and time-varying system, which therefore needs a robust state feedback. The variable structure control theory can be useful for this situation when model uncertainties, parameter variations and disturbances occur. The main contribution of this paper consists in designing a multi-input/multi-output (MIMO) variable structure control of the denitrification process. Two approaches are developed: classical and generalised variable structure control. The performances of the two approaches are compared and illustrated by means of simulations.

Keywords: biological process, nonlinear system, variable structure control

1. INTRODUCTION

Water plays a very significant role in the human life. During last decades, the quality of water is more and more damaged by industrial, domestic and agricultural activities which increase the presence of nitrogen substances. The high concentration of nitrogen contents affects human health. Indeed, their regulation in water becomes more stringent. There are several processes used for the biological treatment of wastewater such as the activated sludge process, the process based on the principle of aerated lagoon and the denitrification process. To maintain the performances and the effectiveness of these processes, the use of adequate control laws proves to be necessary. These processes are nonlinear and time varying systems, which therefore need a robust state feedback. The variable structure theory can be useful for this situation when model uncertainties, parameter variations and disturbances occur. The variable structure control consists in bringing the system on so-called sliding surface in the state space and maintaining it on this surface by using a switching algorithm toward an equilibrium state [1, 2]. Variable structure control is well documented in the literature [3-9].

The main contribution of this paper consists of designing a multi-input multi-output (MIMO) variable structure control law for a denitrification process in order to regulate the nitrate and the acetic acid concentrations at the outlet of the reactor. Two approaches are developed: classic and generalized variable structure control.

The paper is organized as follows: in section 2, the process model is briefly described and the problem statement is given. The two nonlinear control algorithms and their application to the denitrification process are described in sections 3 and 4. In section 5, the efficiencies of the control laws are demonstrated via simulation study. A general conclusion ends the paper.

2. PROCESS MODEL AND PROBLEM STATEMENT

2.1. Process model

The considered process is a continuous-flow denitrification process in which a bacterial culture of *Pseudomonas Denitrificans* occurs: the biomass X starts with consuming the acetic acid S_3 and the nitrate S_1 and rejects some nitrites S_2. Then, it continues to consume the acetic acid but the production of nitrites decreases. The mathematical dynamical model of the process is [10]:

$$\begin{cases} \dot{S}_1 = -y_{11}\mu_1 X + D\left(S_{1in} - S_1\right) \\ \dot{S}_2 = \left(y_{12}\mu_1 - y_{22}\mu_2\right) X + D\left(S_{2in} - S_2\right) \\ \dot{S}_3 = -\left(y_{13}\mu_1 + y_{23}\mu_2\right) X + D\left(S_{3in} - S_3\right) \\ \dot{X} = \left(\mu_1 + \mu_2\right) X - k_d X - DX \end{cases} \quad (1)$$

where S_1, S_2, S_3 and X are the concentrations of the corresponding species; μ_1 and μ_2 represent the specific growth rate of the biomass respectively on the acetic acid and the nitrite; k_d is the coefficient of mortality, S_{1in}, S_{2in} and S_{3in} are the input concentrations respectively of S_1, S_2 and S_3; D is the dilution rate and finally $\{y_{ij}\}$ are yield coefficients.

The expressions of the two specific growth rates μ_1 and μ_2 are given by:

$$\begin{aligned} \mu_1 &= \mu_{1\max}\frac{S_3}{\left(S_3+k_{S_3}\right)}\frac{S_1}{\left(S_1+k_{S_1}\right)} \\ \mu_2 &= \mu_{2\max}\frac{S_3}{\left(S_3+k_{S_3}\right)}\frac{S_2}{\left(S_2+k_{S_2}\right)} \end{aligned} \qquad (2)$$

where $\mu_{1\max}$ and $\mu_{2\max}$ are the maximal values of μ_1 and μ_2; k_{S_1}, k_{S_2} and k_{S_3} are the constants of affinity associated respectively to the nitrate, to the nitrite and to the acetic acid.

2.2. Problem statement

The objective of the process control is to regulate the nitrate and acetic acid concentrations at the outlet of the reactor by acting respectively on the dilution rate *D* and on the inlet acetic acid concentration S_{3in}. The bioreactor to be controlled is a nonlinear and time varying system, which therefore needs a robust state feedback. The variable structure control theory can be useful for this situation when model uncertainties, parameter variations and disturbances occur. For the synthesis of the control law, we consider the following assumptions:

A1: All the parameters of the process model (equations 1 and 2) are known or can be determined by using an estimation technique [11-13].

A2: The nitrate concentration S_1 is bounded: $0 < S_1 < S_{1in}$.

3. MIMO CLASSIC VARIABLE STRUTURE CONTROL (MCVSC)

3.1. Presentation

Let consider the following nonlinear system:

$$\dot{\xi}(t) = f(t,\xi) + g(t,\xi)u(t) \qquad (3)$$

where $\xi(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control vector, $f \in \Re^n$ is a nonlinear vector of ξ and *g* is an $n \times m$ dimensional matrix.

The classic variable structure control law is obtained by forcing each control variable u_i of the control vector to satisfy the following law:

$$u_i = \begin{cases} u_i^+(t,\xi) & \text{if} \quad \sigma_i(\xi) > 0 \\ u_i^-(t,\xi) & \text{if} \quad \sigma_i(\xi) < 0 \end{cases} \quad i = 1,\dots,m \qquad (4)$$

on *m* sliding surfaces of dimension $(n-1)$ designed by $\sigma_i = \{\xi / \sigma_i(t,\xi) = 0\}, i = 1,\dots,m$. It is shown in [1] that the control law u_i can be the sum of two components:

$$u_i = u_{eq_i} + \Delta u_i,\ i = 1,\dots,m \qquad (5)$$

u_{eq_i} is the equivalent control law which is obtained for an ideal sliding mode for which the system state is maintained on the sliding surface:

$$\sigma_i(\xi) = 0,\ i = 1,\dots,m \qquad (6)$$

An ideal sliding mode is ensured only if $\dot{\sigma}_i(\xi) = 0,\ i = 1,\dots,m$

Then

$$\frac{\partial \sigma_i}{\partial \xi}\left[f(t,\xi) + g(t,\xi)u_{eq}\right] = 0,\ i = 1,\dots,m \qquad (7)$$

The equivalent control is then derived:

$$u_{eq_i} = -\left[\left(\frac{\partial \sigma_i}{\partial \xi}\right)^T g(t,\xi)\right]^{-1}\left[\left(\frac{\partial \sigma_i}{\partial \xi}\right)^T f(t,\xi)\right], i = 1,\dots,m \qquad (8)$$

The matrix $\left[\left(\frac{\partial \sigma_i}{\partial \xi}\right)^T g(t,\xi)\right]$ is supposed to be invertible.

Δu_i is the high frequency component defined by

$$\Delta u_i = -K_i\, sgn(\sigma_i(\xi)),\ i = 1,\dots,m \qquad (9)$$

The gains K_i are determined by considering the sliding condition $\sigma_i \dot{\sigma}_i < 0$.

The *sign* function is defined by:

$$sgn(\sigma_i(\xi)) = \begin{cases} 1 & \text{if} \quad \sigma_i(\xi) > 0 \\ 0 & \text{if} \quad \sigma_i(\xi) = 0, \quad i = 1,\dots,m \\ -1 & \text{if} \quad \sigma_i(\xi) < 0 \end{cases} \qquad (10)$$

To remedy the problem of the oscillations of high frequencies due to the switching terms, we can replace the sign function by the Sat function inside a layer limits.

The Sat function is defined as follows:

$$Sat(\sigma_i(\xi)/\psi_i) = \begin{cases} \sigma_i(\xi)/\psi_i & \text{if } \left|\sigma_i(\xi)/\psi_i\right| \le 1 \\ \text{sgn}(\sigma_i(\xi)) & \text{if } \left|\sigma_i(\xi)/\psi_i\right| > 1 \end{cases},\quad i = 1,\dots,m \qquad (11)$$

where ψ_i is the width of the layer limits.

3.2. Application to the denitrification process

In order to decouple the control variables in (1), consider the following intermediate control variables:

$$U_1 = D \qquad (12)$$

and

$$U_2 = DS_{3in} \tag{13}$$

In this case, the denitrification process (1) can be written in the nonlinear form (3) with:

$$\xi = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ X \end{bmatrix}; \qquad f(t,\xi) = \begin{bmatrix} -y_{11}\mu_1 X \\ (y_{12}\mu_1 - y_{22}\mu_2)X \\ -(y_{13}\mu_1 + y_{23}\mu_2)X \\ (\mu_1 + \mu_2 - k_d)X \end{bmatrix};$$

$$g(t,\xi) = \begin{bmatrix} S_{1in} - S_1 & 0 \\ S_{2in} - S_2 & 0 \\ -S_3 & 1 \\ -X & 0 \end{bmatrix} \text{ and } u = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

The outputs y_i are defined by:

$$y_1 = S_1 \tag{14}$$

and

$$y_2 = S_3 \tag{15}$$

The sliding surfaces σ_i are defined as the error between the outputs y_i and the desired values y_{d_i} as follows:

$$\sigma_1 = S_1 - y_{d_1} \tag{16}$$

and

$$\sigma_2 = S_3 - y_{d_2} \tag{17}$$

The equivalent controls u_{eq_i} are derived from (8):

$$u_{eq_1} = \frac{y_{11}\mu_1 X}{S_{1in} - S_1} \tag{18}$$

and

$$u_{eq_2} = \frac{y_{11}\mu_1 X S_3}{S_{1in} - S_1} + (y_{13}\mu_1 + y_{23}\mu_2)X \tag{19}$$

4. MIMO GENERALIZED VARIABLE STRUCTURE CONTROL (MGVSC)

Our objective consists on developing a MGVSC approach. This control law is an extension of an approach presented by Sira Ramirez [8] in the SISO case. The synthesis of the variable structure controller is based on a generalized dynamic represented in a generalized observability or controller canonical form. From this form, rises a dynamic linearizing feedback [14], which can be synthesized according to the theory of the sliding mode [8].

We consider the following nonlinear MIMO system:

$$\begin{cases} \dot{X} = f(X,U) \\ y = h(X) \end{cases} \tag{20}$$

where $X \in \Re^n$ is the state vector, $U \in \Re^m$ is the input vector and $y \in \Re^m$ is the output vector.

4.1. Generalized canonical forms

By using the formalism of the differential algebra, the elimination of state *X* in (20), under certain assumptions, allows to associate the system (20) *j* generalized controller canonical forms (*GCCF*) locally presented by [14]:

$$\begin{cases} \dot{\zeta}_{1,j} = \zeta_{2,j} \\ \dot{\zeta}_{2,j} = \zeta_{3,j} \\ . \\ . \\ \dot{\zeta}_{n-1,j} = \zeta_{n,j} \\ \dot{\zeta}_{n,j} = C_j\left(\boldsymbol{\zeta}_j, u_j, \dot{u}_j, ..., u_j^{(\rho)}\right) \end{cases} \tag{21}$$

where:

$\zeta_j = \left[\zeta_{1,j}\zeta_{2,j}...\zeta_{n,j}\right]^T$, $j = 1...m$

$\zeta_{i,j} = y_j^{(i-1)}$, $i = 2,3,...,n-1$

and C_j are a polynomial forms.

The *j* locals generalized observability canonical forms (*GOCF*), corresponding to (21), are obtained by adding the output equation $y_j = \zeta_{1,j}$.

$$\begin{cases} \dot{\zeta}_{1,j} = \zeta_{2,j} \\ \dot{\zeta}_{2,j} = \zeta_{3,j} \\ . \\ . \\ \dot{\zeta}_{n-1,j} = \zeta_{n,j} \\ \dot{\zeta}_{n,j} = C_j\left(\boldsymbol{\zeta}_j, u_j, \dot{u}_j, ..., u_j^{(\rho)}\right) \\ y_j = \zeta_{1,j} \end{cases} \tag{22}$$

4.2. Linearizing feedback with variable structure

From the generalized canonical form (*GCCF* or *GOCF*), a linearizing feedback can be obtained as follows [14]:

$$\dot{\zeta}_{n,j} = C_j\left(\boldsymbol{\zeta}_j, u_j, \dot{u}_j, ..., u_j^{(\rho)}\right) = \sum_{i=1}^{n} d_{i,j}\zeta_{i,j} + bv \tag{23}$$

where v indicates a new input.

The search for a solution of this equation results in a linearizing control law which will depend on the type of the used linearizing feedback.

In the next of this work, we consider a variable structure linearizing feedback [8, 15, 16]. For that, we must define *j* adequate sliding surfaces. Let us suppose that these surfaces are linear compared to the vector of state ζ_j

$$\boldsymbol{\sigma}_j = \mathbf{P}^T \boldsymbol{\zeta}_j = \sum_{i=1}^{n} p_{i,j} \boldsymbol{\zeta}_{i,j} \quad (24)$$

where $\{p_{i,j}\}$ represent the coefficients of the sliding surfaces, with $p_{n,j} = 1$.

The sliding functions $\boldsymbol{\sigma}_j$ are introduced into the feedback equation according to the following proposition [8]:

Proposition:
Consider the discontinuous controlled system:

$$\dot{\sigma}_j = -\eta_j \sigma_j + v_j \quad (25)$$

where the variables v_j act as an external control input. Let choose the discontinuous feedback control policy as:

$$v_j = -\eta_j W_j \operatorname{sgn}(\sigma_j) \quad (26)$$

where $\boldsymbol{\eta}_j$ and W_j are strictly positive quantities.

Then, v_j globally create a sliding regime on $(\boldsymbol{\sigma}_j = 0)$.

Furthermore, any trajectory starting on the value $\sigma_j = \sigma_j(0)$, at time $t = 0$, reaches the condition $(\boldsymbol{\sigma}_j = 0)$ in finite time T_j, given by:

$$T_j = \boldsymbol{\eta}_j^{-1} \, ln\left(1 + \frac{\boldsymbol{\sigma}_j(0)}{W_j}\right).$$

A dynamical variable structure feedback controller is readily obtained for the dynamical system (20) if we impose on the evolution of the auxiliary output variable $\boldsymbol{\sigma}_j$ the discontinuous dynamics considered in the above proposition. From (22-26) one obtains:

$$C_j\left(\boldsymbol{\zeta}_j, u_j, \dot{u}_j, \ldots, u_j^{(p)}\right) = -\sum_{i=1}^{n-1} p_{i,j} \zeta_{i+1,j} - \boldsymbol{\eta}_j \left(\sum_{i=1}^{n} p_{i,j} \zeta_{i,j} + W_j sgn(\boldsymbol{\sigma}_j)\right) \quad (27)$$

which is to be viewed as an implicit scalar differential equation with discontinuous right-hand-side.

When the sliding mode is reached ($\sigma_j = 0$ and $\dot{\sigma}_j = 0$) the dynamics of the system (22) becomes in the following reduced order system:

$$\begin{cases} \dot{\zeta}_{1,j} = \zeta_{2,j} \\ \dot{\zeta}_{2,j} = \zeta_{3,j} \\ . \\ \dot{\zeta}_{n-1,j} = -\sum_{i=1}^{n-1} p_{i,j} \zeta_{i,j} \end{cases} \quad (28)$$

The stability of this system is ensured by a suitable choice of the coefficients $\{p_{i,j}\}$ such as the coefficients of a Hurwitz polynomial.

The advantage of the presented approach in this paper comes owing to the fact that in the expression of the control law (equation 27), the commutation is done on the highest derivative of the inputs u_j. This results in a sliding mode characterized by a discontinuity on the highest derivative of the control law. This fact rises an interesting property: the control law is characterized by soft actions, because it is obtained from *p* integrations, thus reducing the phenomenon of "chattering".

4.3. Application to the process of idenitrification

For the synthesis of the control laws, we define a generalized canonical form for each output as follows:

$$\begin{cases} \zeta_{11} = S_1 - y_{d1} \\ \zeta_{21} = \dot{\zeta}_{11} = \dot{S}_1 \\ \zeta_{31} = \dot{\zeta}_{21} = \ddot{S}_1 \\ \zeta_{41} = \dot{\zeta}_{31} = \dddot{S}_1 \end{cases} \quad (29)$$

and

$$\begin{cases} \zeta_{12} = S_3 - y_{d2} \\ \zeta_{22} = \dot{\zeta}_{12} = \dot{S}_3 \\ \zeta_{32} = \dot{\zeta}_{22} = \ddot{S}_3 \\ \zeta_{42} = \dot{\zeta}_{32} = \dddot{S}_3 \end{cases} \quad (30)$$

By developing calculations, we obtain the following expressions for $\dot{\zeta}_{41}$ and $\dot{\zeta}_{42}$.

$$\dot{\zeta}_{41} = C_1\left(\zeta_{11}, \zeta_{21}, \zeta_{31}, \zeta_{41}, D, \dot{D}, \ddot{D}, \dddot{D}\right) \quad (31)$$

$$\dot{\zeta}_{42} = C_2\left(\zeta_{12}, \zeta_{22}, \zeta_{32}, \zeta_{42}, S_{3in}, \dot{S}_{3in}, \ddot{S}_{3in}, \dddot{S}_{3in}, D, \dot{D}, \ddot{D}, \dddot{D}\right) \quad (32)$$

Then, the generalized observability canonical forms are written:

$$\begin{cases} \dot{\zeta}_{11} = \zeta_{21} \\ \dot{\zeta}_{21} = \zeta_{31} \\ \dot{\zeta}_{31} = \zeta_{41} \\ \dot{\zeta}_{41} = C_1\left(\zeta_{11}, \zeta_{21}, \zeta_{31}, \zeta_{41}, D, \dot{D}, \ddot{D}, \dddot{D}\right) \\ y_1 = \zeta_{11} \end{cases} \quad (33)$$

and

$$\begin{cases} \dot{\zeta}_{12} = \zeta_{22} \\ \dot{\zeta}_{22} = \zeta_{32} \\ \dot{\zeta}_{32} = \zeta_{42} \\ \dot{\zeta}_{42} = C_2\left(\zeta_{12},\zeta_{22},\zeta_{32},\zeta_{42},S_{3in},\dot{S}_{3in},\ddot{S}_{3in},\dddot{S}_{3in},D,\dot{D},\ddot{D},\dddot{D}\right) \\ y_2 = \zeta_{12} \end{cases} \quad (34)$$

Consider the following sliding surfaces σ_1 and σ_2:

$$\sigma_1(\zeta_1) = \zeta_{41} + p_{31}\zeta_{31} + p_{21}\zeta_{21} + p_{11}\zeta_{11} \quad ,p_{11},p_{21},p_{31} > 0 \quad (35)$$

$$\sigma_2(\zeta_2) = \zeta_{42} + p_{32}\zeta_{32} + p_{22}\zeta_{22} + p_{12}\zeta_{12} \quad ,p_{12},p_{22},p_{32} > 0 \quad (36)$$

Imposing on σ_j the asymptotically stable discontinuous controlled dynamics defined by:

$$\dot{\sigma}_1 + \eta_1\sigma_1 = -\eta_1 W_1\, sgn(\sigma_1) \quad (37)$$

$$\dot{\sigma}_2 + \eta_2\sigma_2 = -\eta_2 W_2\, sgn(\sigma_2) \quad (38)$$

one readily obtains the following linearizing feedbacks.

$$C_1 = -p_{11}\zeta_{21} - p_{21}\zeta_{31} - p_{31}\zeta_{41} - \eta_1\left(\sigma_1 + W_1\, sgn(\sigma_1)\right) \quad (39)$$

$$C_2 = -p_{12}\zeta_{22} - p_{22}\zeta_{32} - p_{32}\zeta_{42} - \eta_2\left(\sigma_2 + W_2\, sgn(\sigma_2)\right) \quad (40)$$

These equations provide us the following expressions of the control laws:

$$\ddot{D} = \left(S_{1in} - S_1\right)^{-1} \begin{bmatrix} -p_{11}\zeta_{21} - p_{21}\zeta_{31} - p_{31}\zeta_{41} - \eta_1\left(\sigma_1 + W_1 sgn(\sigma_1)\right) \\ + y_{11}\mu_1\ddot{X} + 3y_{11}\left(H_1\dot{S}_1 + H_2\dot{S}_3\right)\dot{X} + 3y_{11}\dot{X}A \\ + y_{11}X\left[B + C + E\right] + 3\ddot{D}\dot{S}_1 + 3\dot{D}\ddot{S}_1 + D\dddot{S}_1 \end{bmatrix} \quad (41)$$

with:

$$A = H_1\ddot{S}_1 + H_2\ddot{S}_3 + \dot{S}_1\left(K_1\dot{S}_1 + K_2\dot{S}_3\right) + \dot{S}_3\left(K_3\dot{S}_1 + K_4\dot{S}_3\right)$$

$$B = H_1\dddot{S}_1 + H_2\dddot{S}_3 + 2\ddot{S}_1\left(K_1\dot{S}_1 + K_2\dot{S}_3\right) + 2\ddot{S}_3\left(K_3\dot{S}_1 + K_4\dot{S}_3\right)$$

$$C = \dot{S}_1\left[K_1\ddot{S}_1 + K_2\ddot{S}_3 + \dot{S}_1\left(L_1\dot{S}_1 + L_2\dot{S}_3\right) + \dot{S}_3\left(L_3\dot{S}_1 + L_4\dot{S}_3\right)\right]$$

$$E = \dot{S}_3\left[K_3\ddot{S}_1 + K_4\ddot{S}_3 + \dot{S}_1\left(L_5\dot{S}_1 + L_6\dot{S}_3\right) + \dot{S}_3\left(L_7\dot{S}_1 + L_8\dot{S}_3\right)\right]$$

and

$$\dddot{S}_{3in} = D^{-1} \begin{bmatrix} -p_{12}\zeta_{22} - p_{22}\zeta_{32} - p_{32}\zeta_{42} - \eta_2\left(\sigma_2 + W_2 sgn(\sigma_2)\right) \\ + expr1 + expr2 + expr3 + expr4 + 3\dot{D}\ddot{S}_3 + 3\ddot{D}\dot{S}_3 \\ + D\dddot{S}_3 + \dddot{D}S_3 - 3\ddot{S}_{3in}\dot{D} - 3\dot{S}_{3in}\ddot{D} - \dddot{D}S_{3in} \end{bmatrix} \quad (42)$$

with:

$$expr1 = -\left(y_{13}\mu_1 + y_{23}\mu_2\right)\ddot{X} - 3y_{13}\dot{X}\left(H_1\dot{S}_1 + H_2\dot{S}_3\right) - 3y_{23}\dot{X}\left(H_3\dot{S}_2 + H_4\dot{S}_3\right)$$

$$expr2 = -3\dot{X}\left(y_{13}A + y_{23}F\right)$$

$$expr3 = -y_{13}X\left(B + C + E\right)$$

$$expr4 = -y_{23}X\left(G + H + I\right)$$

$$G = H_3\dddot{S}_2 + H_4\dddot{S}_3 + 2\ddot{S}_2\left(K_5\dot{S}_2 + K_6\dot{S}_3\right) + 2\ddot{S}_3\left(K_7\dot{S}_2 + K_8\dot{S}_3\right)$$

$$H = \dot{S}_2\left[K_5\ddot{S}_2 + K_6\ddot{S}_3 + \dot{S}_2\left(L_9\dot{S}_2 + L_{10}\dot{S}_3\right) + \dot{S}_3\left(L_{11}\dot{S}_2 + L_{12}\dot{S}_3\right)\right]$$

$$I = \dot{S}_3\left[K_7\ddot{S}_2 + K_8\ddot{S}_3 + \dot{S}_2\left(L_{13}\dot{S}_2 + L_{14}\dot{S}_3\right) + \dot{S}_3\left(L_{15}\dot{S}_2 + L_{16}\dot{S}_3\right)\right]$$

$$H_1 = \frac{\partial\mu_1}{\partial S_1}, \quad H_2 = \frac{\partial\mu_1}{\partial S_3}, \quad H_3 = \frac{\partial\mu_2}{\partial S_2}, \quad H_4 = \frac{\partial\mu_2}{\partial S_3},$$

$$K_1 = \frac{\partial H_1}{\partial S_1}, \quad K_2 = \frac{\partial H_1}{\partial S_3}, \quad K_3 = \frac{\partial H_2}{\partial S_1}, \quad K_4 = \frac{\partial H_2}{\partial S_3},$$

$$K_5 = \frac{\partial H_3}{\partial S_2}, \quad K_6 = \frac{\partial H_3}{\partial S_3}, \quad K_7 = \frac{\partial H_4}{\partial S_2}, \quad K_8 = \frac{\partial H_4}{\partial S_3},$$

$$L_1 = \frac{\partial K_1}{\partial S_1}, \ L_2 = \frac{\partial K_1}{\partial S_3}, \ L_3 = \frac{\partial K_2}{\partial S_1}, \ L_4 = \frac{\partial K_2}{\partial S_3}, \ L_5 = \frac{\partial K_3}{\partial S_1},$$

$$L_6 = \frac{\partial K_3}{\partial S_3}, \quad L_7 = \frac{\partial K_4}{\partial S_1}, \quad L_8 = \frac{\partial K_4}{\partial S_3}, L_8 = \frac{\partial K_5}{\partial S_2},$$

$$L_{10} = \frac{\partial K_5}{\partial S_3}, \quad L_{11} = \frac{\partial K_6}{\partial S_2}, \quad L_{12} = \frac{\partial K_6}{\partial S_3}, \quad L_{13} = \frac{\partial K_7}{\partial S_2},$$

$$L_{14} = \frac{\partial K_7}{\partial S_3}, \ L_{15} = \frac{\partial K_8}{\partial S_2}, \ L_{16} = \frac{\partial K_8}{\partial S_3}$$

5. SIMULATION RESULTS

5.1. Numerical values

The simulation of the model and the controllers is run with typical values of kinetic parameters and initial conditions given in Table 1.

Table 1: Initial conditions and parameter values

Variables	Values	Parameters	Values	Parameters	Values
$S_1(0)$	$0.6g/l$	S_{1in}	$1g/l$	$\mu_{1\max}$	$0.17h^{-1}$
$S_2(0)$	$0g/l$	y_{11}	6.2	$\mu_{2\max}$	$0.085h^{-1}$
$S_3(0)$	$2.77g/l$	y_{12}	3.3	k_{S_1}	$0.05g/l$
$X(0)$	$0.15g/l$	y_{22}	1.2	k_{S_2}	$0.07g/l$
$D(0)$	$0.6h^{-1}$	y_{13}	1.1	k_{S_3}	$0.1g/l$
$S_{3in}(0)$	$0g/l$	y_{23}	1.6	k_d	$0.025h^{-1}$

To test the robustness of the controllers, we introduced disturbances on the kinetic parameters of the system. These disturbances are presented in Table 2.

Table 2: Parameter changes

Time (h)	Parameter changes	Time (h)	Parameter changes
100	$\mu_{2\max} = 0.1h^{-1}$	300	$k_{S_2} = 0.1g/l$

The setpoint changes of the controlled variables and the design parameters of the controller and the estimator are given in the following tables:

Table 3: Controlled variables setpoints

$y_{d_1} = 0.1g/l$ for $0 \le t < 200$ and $0.03g/l$ for $200 \le t \le 400$

$y_{d_2} = 2.5g/l$ for $0 \le t < 200$ and $3.5g/l$ for $200 \le t \le 400$

Table 4: Controller parameters

MCVSC	MGVSC
$K_1 = K_2 = 0.01$ $\psi_1 = \psi_2 = 0.04$	$p_{11} = p_{12} = 1, p_{21} = p_{22} = 3, p_{31} = p_{32} = 3,$ $\eta_1 = \eta_2 = 0.1, W_1 = W_2 = 0.5$

5.2. Results comment

The simulation results are illustrated in Fig 1 to Fig 6 for the classic approach and in Fig 7 to Fig 12 for the generalized approach.

The output variables that are the nitrate and the acetic acid concentrations and their corresponding reference trajectories are given in Fig 1 and 4 for the MCVSC and in Fig 7 and 11 for the MGVSC. These figures show the performance of the regulators. In particular, one can appreciate the ability of the controllers to track the desired values of the controlled variables in response to the step change of the setpoints. The introduced perturbations, due to the kinetic parameter variations, did not vary the profiles of the controlled variables in the case of the MCVSC, whereas for the MGVSC these perturbations are rejected after 10h. This situation is explained by the fact that these perturbations occur directly in the expression of the MCVSC and in the third derivative of the MGVSC.

The evolutions of the control variables, which are the dilution rate *D* and the inlet acetic acid concentration S_{3in}, are shown in Fig 2 and Fig 5 for the MCVSC, and in Fig 8 and Fig 11 for the MGVSC. These profiles show the reaction to changes due to abrupt jumps of the kinetic parameters of the process and step changes in the setpoints. In particular, the MGVSC has a soft profile during the setpoint change of the controlled variable S_3 (cf. Fig 11). The MCVSC is characterized by discontinuous controls. Indeed, the use of the saturation function (Sat) enabled us to have a less active control due to the parameter ψ_i. A low value of ψ_i is equal to an agitated control, whereas a high value generates a significant regulation error.

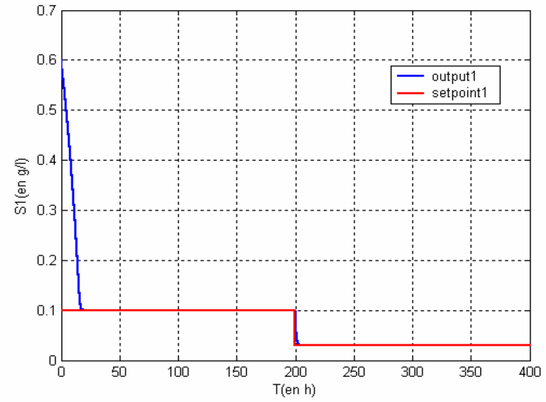

Fig 1: Evolution of S_1 with MCVSC

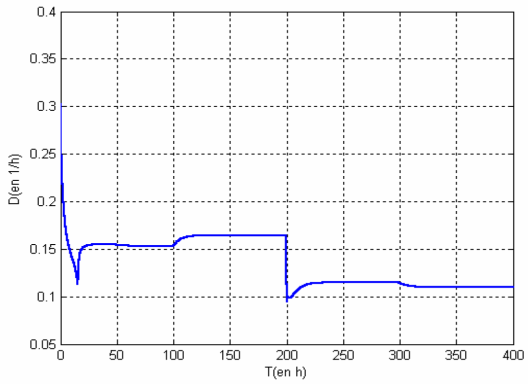

Fig 2: Evolution of D with MCVSC

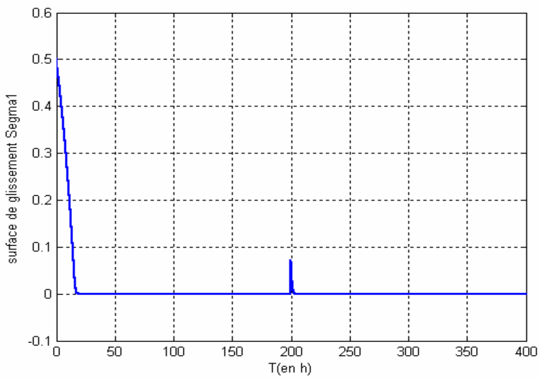

Fig 3: Evolution of σ_1 with MCVSC

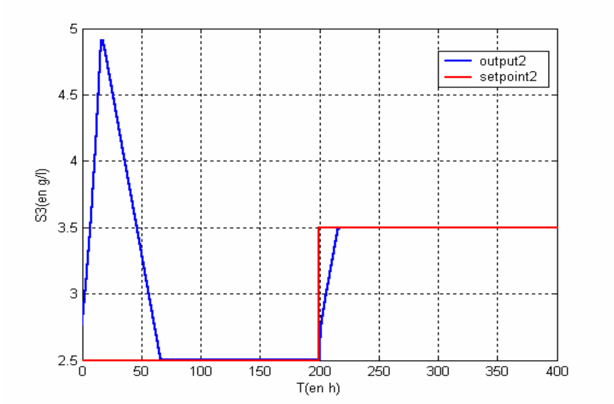

Fig 4: Evolution of S_3 with MCVSC

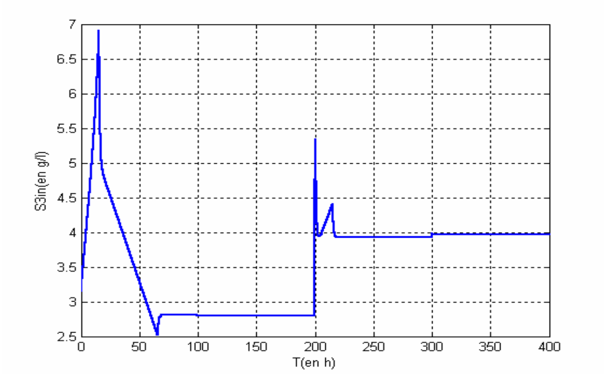

Fig 5: Evolution of S_{3in} with MCVSC

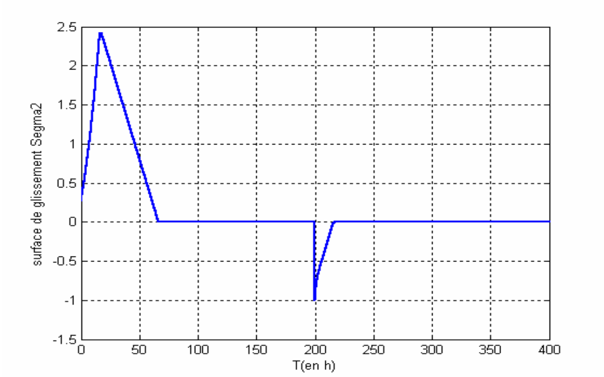

Fig 6: Evolution of σ_2 with MCVSC

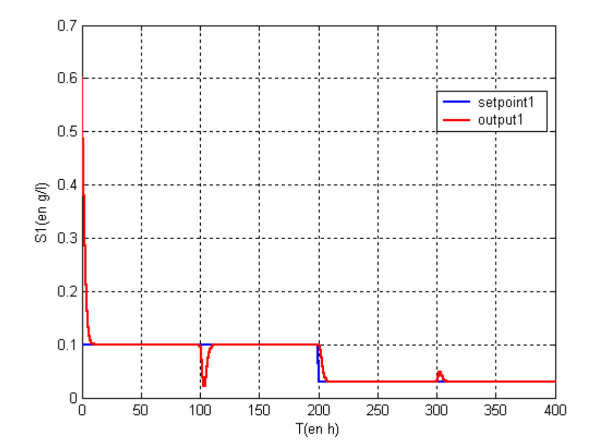

Fig 7: Evolution of S_1 with MGVSC

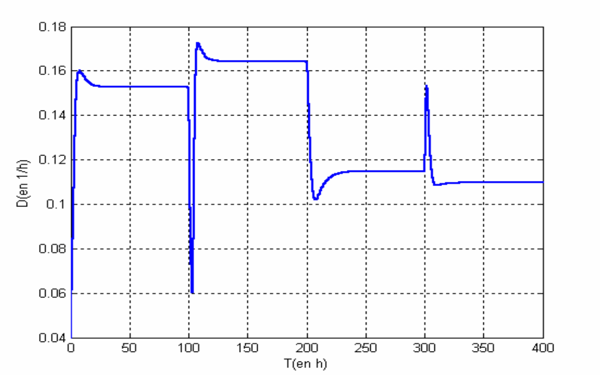

Fig 8: Evolution of D with MGVSC

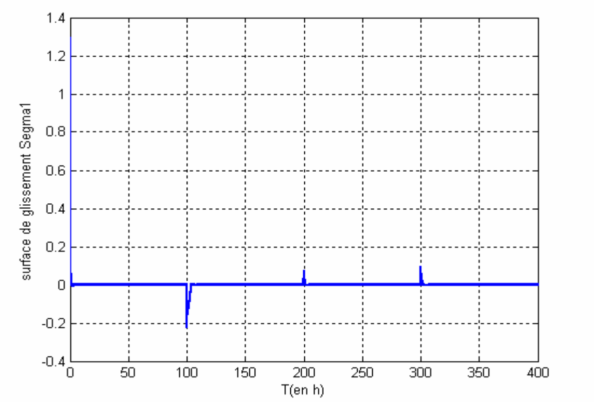

Fig 9: Evolution of σ_1 with MGVSC

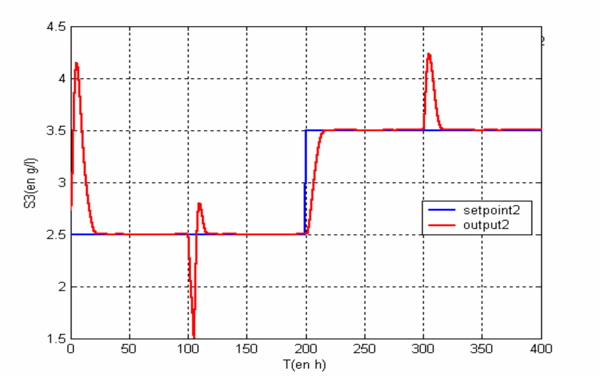

Fig 10: Evolution of S_3 with MGVSC

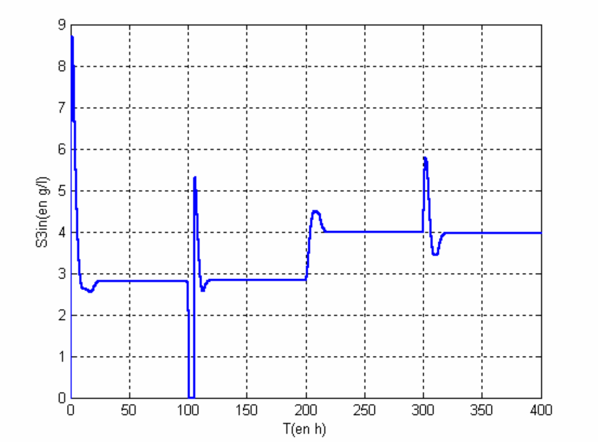

Fig 11: Evolution of S_{3in} with MGVSC

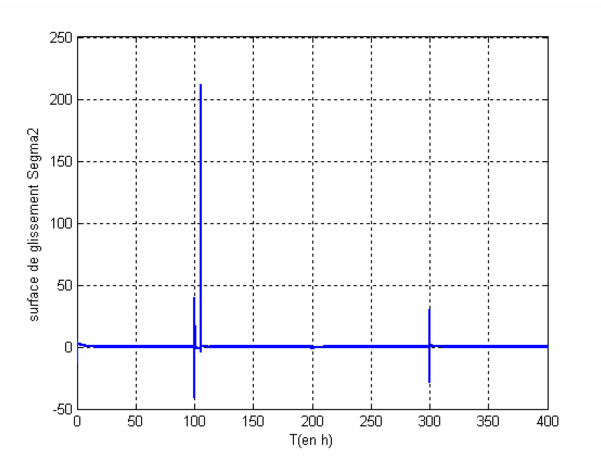

Fig 12: Evolution of σ_2 with MGVSC

6. CONCLUSION

This paper introduces the MIMO variable structure control in order to regulate the nitrate and the acetic acid concentrations at desired values by acting respectively on the dilution rate and on the inlet acetic acid concentration at the outlet of a denitrification process. Two approaches are designed: classic and generalized variable structure control. For the classic approach, the control law is the sum of low and high frequency components. For the generalized approach, the controller is based on a generalized observability canonical form. The robustness of the control laws are compared in the simulation results.

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Ì Í Î Ã Î Ì ÅÐ Í Î Ó Ï ÐÀÂËÅ Í ÈÅ Ñ Ï Ð Î Ì Å Í ËÈÂÀ ÑÒÐÓÊÒÓÐÀ Í À Ï Ð Î ÖÅÑ Í À Ï ÐÅ×ÈÑÒÂÀ Í Å Í À Î Ò Ï À Ä Í È Â Î ÄÈ

Ê. Äàåø, Ò. Äàìàê, À. Òóìè

Ðåçþìå: Â ñòàòèÿòà ñå ðàçãëåæäà ïðîáëåì çà óïðàâëåíèå íà íåëèíåéíà ñèñòåìà, â êîÿòî îáåêòúò çà óïðàâëåíèå ïðåäñòàâëÿâà äåíèòðèôèêàöèîíåí ïðîöåñ çà áèîëîãè÷íî ïðå÷èñòâàíå íà îòïàäíè âîäè. Óïðàâëÿâàíèÿò áèîðåàêòîð å íåëèíåéíà è íåñòàöèîíàðíà ñèñòåìà, ïðè êîÿòî å íåîáõîäèìà ðîáàñòíà îáðàòíà âðúçêà ïî ñúñòîÿíèåòî. Òåîðèÿòà íà ñèñòåìèòå ñ ïðîìåíëèâà ñòðóêòóðà (ÑÏÑ) èìà ãîëÿìà ïðèëîæèìîñò ïðè íàëè÷èå íà íåîïðåäåëåíîñòè â ìîäåëà, ïàðàìåòðè÷íè ñìóùåíèÿ è âàðèàöèè. Îñíîâíèÿò ïðèíîñ íà ñòàòèÿòà ñå ñúñòîè â ïðîåêòèðàíåòî íà ìíîãîìåðíà ÑÏÑ çà óïðàâëåíèå íà äå-íèòðèôèêàöèîííèÿ ïðîöåñ. Èçïîëçâàíè ñà äâà ïîäõîäà: êëàñè÷åñêî è îáîáùåíî óïðàâëåíèå ñ ïðî-ìåíëèâà ñòðóêòóðà. Êà÷åñòâàòà íà äâàòà ïîäõîäà ñà ñðàâíåíè è èëþñòðèðàíè ÷ðåç ñèìóëàöèîííè èçñëåäâàíèÿ.