

II. SPACE TECHNOLOGY AND ENVIRONMENTAL MONITORING

TOPOGRAPHY ANALYSIS OF TERRAIN WITHIN PROTONILUS MENSAE REGION OF MARS USING FRACTAL DIMENSION AND LACUNARITY

Rosen Iliev, Boyko Rangelov

Space Research and Technology Institute, Bulgarian Academy of Sciences, Sofia, Bulgaria

Abstract: This study investigates the surface complexity of the Protonilus Mensae region in the northern hemisphere of Mars using an integrated approach based on fractal dimension (FD) and lacunarity analysis. Digital elevation model (DEM) data derived from the Mars Orbiter Laser Altimeter (MOLA), with a spatial resolution of 463 m, were analyzed using Python-based algorithms. Surface roughness was analyzed using a variogram-based fractal method. The results show that the topography of Protonilus Mensae exhibits fractal behavior, with FD values ranging from 2.11 to 2.70 and a mean value of 2.35. Spatial patterns of FD reveal predominantly low to moderate surface complexity. Lacunarity analysis was employed to characterize spatial heterogeneity and the distribution of topographic gaps. (Lacunarity means empty spaces (gaps) in DEM with similar fractal dimensions but different surface view.) The results show a strong dependence of lacunarity on elevation, with maximum heterogeneity occurring at intermediate elevations associated with fretted (not smooth) terrain. Lower and higher elevations exhibit reduced lacunarity. The combined fractal and lacunarity results suggest that the present-day morphology of Protonilus Mensae represents a degraded landscape shaped by a balance between roughness-enhancing processes and smoothing processes. This integrated quantitative approach provides new insights into the geomorphological evolution of not flat terrains on Mars.

Keywords: Protonilus Mensae, Mars, fractal dimension, lacunarity, variogram, smooth terrains.

INTRODUCTION

The Protonilus Mensae region is a broad and complex landscape in the northern hemisphere of Mars, located in the Ismenius Lacus quadrangle at approximately 43.9° N latitude and 49.4° E

longitude (Fig.1). It lies along the transitional boundary between the highly cratered southern highlands and the low-lying northern plains, a global contrast commonly referred to as the Martian dichotomy.

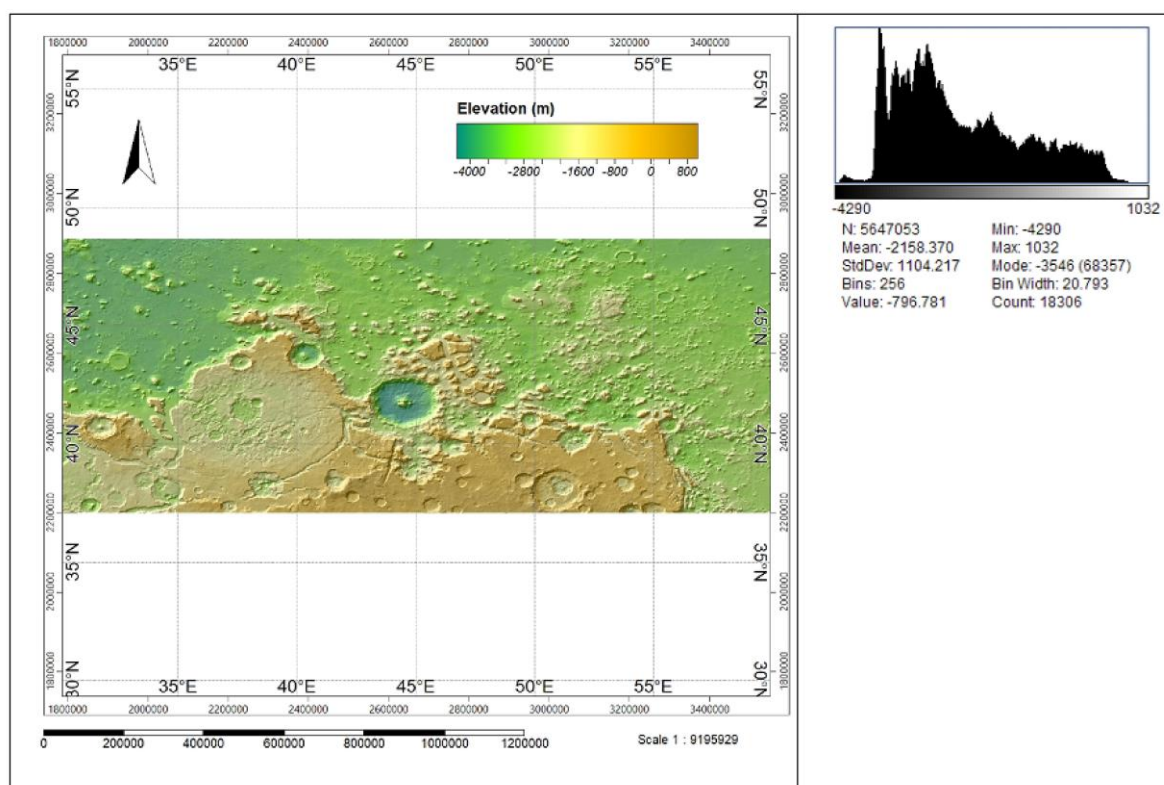


Fig. 1. 3D surface map of Protonilus Mensae region of Mars with the vertical elevation histogram (right panel)



doi.org/10.32006/eeep.2026.1.3037

This article is distributed under the terms of the Creative Commons Attribution License

The landforms of Protonilus Mensae are classified as fretted terrain (Sharp, 1973), characterized by a mixture of mesas, steep escarpments, and broad, flat valleys that have been incised and reshaped by erosional processes.

Mesas generally exhibit flat tops and steep sides, reflecting resistance to erosion relative to the surrounding material. The valleys between mesas have wide, flat floors and steep walls, showing significant vertical relief over short horizontal distances. This heterogeneous landscape reflects a complex history of surface modification, including glacial activity (Sharp, 1973; Madeleine et al., 2007; Soare et al., 2022).

Morphological evidence, such as lobate debris aprons and lineated valley fill (Morgan et al., 2009; Baker et al., 2010), indicates past ice-rich flows that have shaped the region's relief. These glacially influenced features formed where ice-laden material flowed down slopes and valleys, leaving patterned surfaces and ridged outlines.

To quantitatively characterize such complex terrain, fractal analysis provides a powerful mathematical tool. The fractal dimension measures how surface changes across spatial scales, allowing comparison of terrain roughness. Previous studies have applied fractal methods to various Martian landforms: Ching et al. (1993) analyzed craters, Stepinski et al. (2002) examined drainage basins, and Turcotte (1987), Landais et al. (2015), Delière et al. (2016), Demin et al. (2017), and Pardo-Iguzquiza and Dowd (2022) employed fractal approaches for regional and global topography. More recently, Iliev and Rangelov (2020a) investigated mesas in Deuteronilus Mensae, while Iliev and Rangelov (2025b) used fractal dimension to study the spatial distribution of some landform types across both hemispheres.

Lacunarity, a complementary metric, quantifies the texture of spatial patterns by describing the distribution of gaps or voids. Applied to Mars, lacunarity captures the “gappiness” of fretted terrain, such as the arrangement of valleys relative to highland areas.

By combining fractal dimension and lacunarity, researchers gain a more comprehensive view of topography. Fractal dimension quantifies the

scaling of roughness, while lacunarity characterizes spatial heterogeneity and gap distribution. Together, these metrics allow detailed, multiscale assessments of terrain complexity in Protonilus Mensae and provide a basis for comparing topographic patterns across different Martian regions, offering insights into the geological processes that have shaped the planet's surface.

METHODS AND DATA

Fractal Analysis of Protonilus Mensae Topography Using Variogram-Based Method

This study employs a variogram-based fractal approach (Mark and Aronson, 1984; Gallant et al., 1994) to characterize the surface roughness and multiscale topographic complexity of the Protonilus Mensae region of Mars. The method is implemented using a Python script within the QGIS environment.

The analytical procedure includes sliding-window technique applied to the DEM data to assess spatial heterogeneity. Square windows of 51×51 pixels (~ 23 km) are moved across the dataset in steps of 25 pixels (~ 12 km). Within each window, a local variogram is computed, capturing elevation differences in horizontal, vertical, and diagonal directions.

Prior to variogram computation, each window is detrended by fitting and removing a best-fit plane. The slope of the log-log plot of semivariance versus lag distance is then used to calculate the fractal dimension (FD) according to the formula:

$$FD = 3 - (\text{slope} / 2) \quad (1)$$

Fractal dimension values are constrained between 2.0 (smooth surface) and 3.0 (highly rough surface).

The approach produces two georeferenced outputs:

- 1) FD map, depicting local surface roughness across Protonilus Mensae.
- 2) R^2 map, indicating the goodness-of-fit of the variogram-based fractal model.

Each pixel in the output represents the centre of the corresponding sliding window, preserving spatial correspondence with the original DEM.

Summary statistics (mean, standard deviation, minimum, and maximum) are computed for all valid FD values. Histograms visualize the distribution of fractal dimensions, enabling comparison across study area.

Lacunarity Analysis of Protonilus Mensae Using Python in QGIS

The study uses a lacunarity-based approach (Mandelbrot, 1983; Plotnick et al, 1993; Allain and Cloitre, 1991; Voss, 1986) to assess the spatial heterogeneity and topography complexity of the Protonilus Mensae region. The analysis is implemented as a Python script within the QGIS environment.

To efficiently compute local statistics for lacunarity, the script constructs integral images of both the elevation values and their squared values. This enables rapid calculation of sums over moving boxes of various sizes and supports robust treatment of missing data (NaN values).

Lacunarity is computed over multiple box sizes (default: 3, 9, 21, 45 pixels) within each sliding window. For each box, the mean and variance of the elevation sums are calculated, and lacunarity is estimated as the ratio of the second moment to the squared first moment:

$$\text{Lacunarity } \Lambda = (\text{mean of squares}) / (\text{square of mean}) \quad (2)$$

Only boxes with a sufficient proportion of valid data (>50% of pixels) contribute to the final lacunarity estimate. This method captures the degree of spatial gaps or heterogeneity in the terrain, supplementing fractal dimension metrics.

Data and software

The topography of Protonilus Mensae region was analyzed using a digital elevation model (DEM) derived from the Mars Orbiter Laser Altimeter (MOLA) (Smith et al., 2001), an instrument onboard NASA's Mars Global Surveyor (MGS) spacecraft (Albee et al., 2001). The DEM data are provided in GeoTIFF format with a spatial resolution of 463 m per pixel. Topographic analyses were conducted using open-source Geographic Information System (GIS) software, specifically QGIS and SAGA-GIS (Conrad et al., 2015).

RESULTS AND DISCUSSION

The results of this research are quantitatively summarized in Table 1 and graphically illustrated in Figures 2, 3 and 4. The interpretations and conclusions follow further.

Table 1. Fractal Dimension (FD) and Coefficient of Determination (R^2) of study area

	Min	Max	Mean	Mode	StdDev
FD	2.114	2.698	2.352	2.385	0.0961
R^2	0.515	0.999	0.959	0.985	0.0343

At first glance, it becomes clear that the topography of Protonilus Mensae has a fractal distribution, with a mean fractal dimension (FD) of 2.352. This is further supported by the very high coefficient of determination (R^2 mean = 0.959), indicating that the power-law scaling is well defined across topography.

Spatially (Fig.2), the fractal dimension map reveals a predominantly low to moderate degree of surface complexity across the region. FD values range from approximately 2.1 to 2.7, however, most of the study area is characterized by intermediate values clustered around the

regional FD mean. This narrow distribution suggests low spatial anisotropy and indicates that surface roughness is relatively uniform in different directions. Areas exhibiting very high fractal dimensions (>2.5), which would correspond to strongly rugged and highly multiscale terrain, are limited in extent and appear as isolated patches rather than continuous domains.

The spatial organization of FD values is non-random and displays clear patterns. Lower FD values (≈ 2.1 – 2.3) tend to correspond to smoother surfaces, associated with gently

sloping plateaus and crater bottoms. In contrast, higher FD values (≈ 2.4 – 2.5) are spatially associated with transition zones between mesas

and valleys, where surface dissection and terrain variability increase.

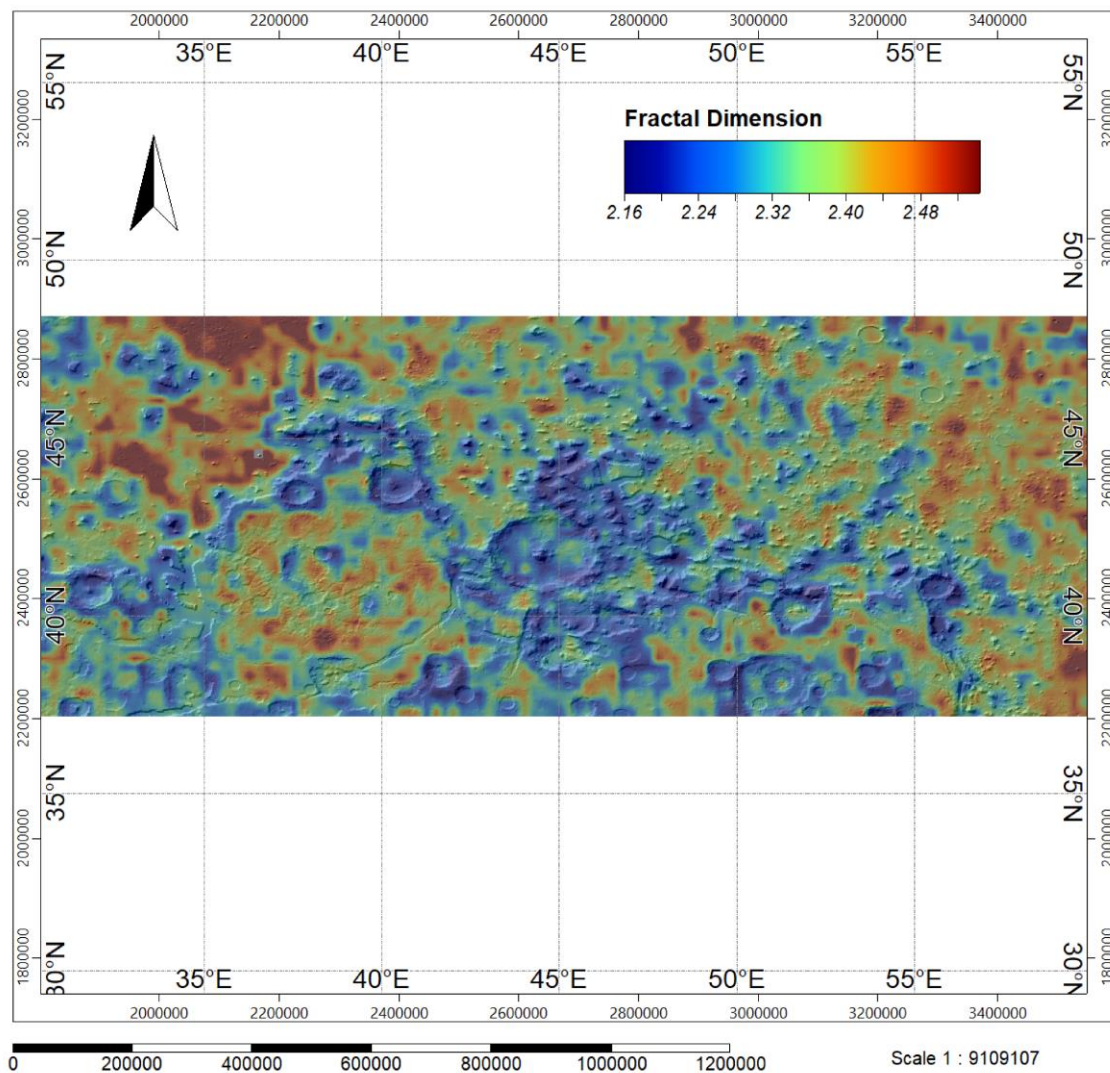


Fig. 2. Fractal Dimension (FD) spatial distribution within Protonilus Mensae region. For better visual perception, the vertical map scale is exaggerated 3 times (x3)

Importantly, the absence of extensive zones with very high FD suggests that, despite his fretted terrain morphology, Protonilus Mensae is not dominated by extremely rough or chaotic topography. This observation supports interpretations invoking ice-related processes that promote both dissection and subsequent surface homogenization over geological history.

The present-day surface morphology reflects a balance between processes that enhance roughness (e.g., valley incision and scarp retreat) and those that reduce it (e.g.,

mantling, creep, and infill), producing a relatively uniform fractal signature across much of the region.

The lacunarity analysis (Fig.3) reveals a clear dependence of surface heterogeneity on elevation within the Protonilus Mensae region. Mean lacunarity (λ) varies across the examined elevation range (appr. -4300 m to $+1000$ m), indicating changes in spatial organization and texture of the surface.

At the lowest elevations (below ~ -3800 m), lacunarity values are relatively low ($\lambda \approx 1.15$ –

1.30), suggesting homogeneous terrain. These areas likely correspond to smoother plains or infilled basins (crater bottoms) where depositional or erosional processes have reduced small-scale variability in topography.

A pronounced increase in lacunarity is observed between ~ -3600 m and -3000 m, where λ reaches higher values of ~ 1.75 – 1.78 . This elevation interval coincides with the highest pixel counts per bin, indicating that the signal is statistically robust. Higher lacunarity in this elevation range suggests strongly heterogeneous surface. Such heterogeneity may reflect the superposition of mesas, troughs, and valleys, potentially shaped by ice-driven mass movement and/or sublimation-driven erosion.

Above ~ -2800 m, mean lacunarity decreases gradually and stabilizes around $\lambda \approx 1.45$ – 1.50 across mid-elevations (approximately -2000 m to -500 m). This plateau suggests a transition to terrain with more uniform spacing of topographic elements, possibly reflecting more advanced degradation of mesas.

At the highest hypsometric elevations (above $+500$ m), lacunarity declines further ($\lambda \approx 1.35$ – 1.40), accompanied by a sharp reduction in areal widespread. This trend is consistent with increasingly homogeneous summit surfaces or gently sloping uplands, where terrain amplitude and spatial clustering of local landforms are diminished.

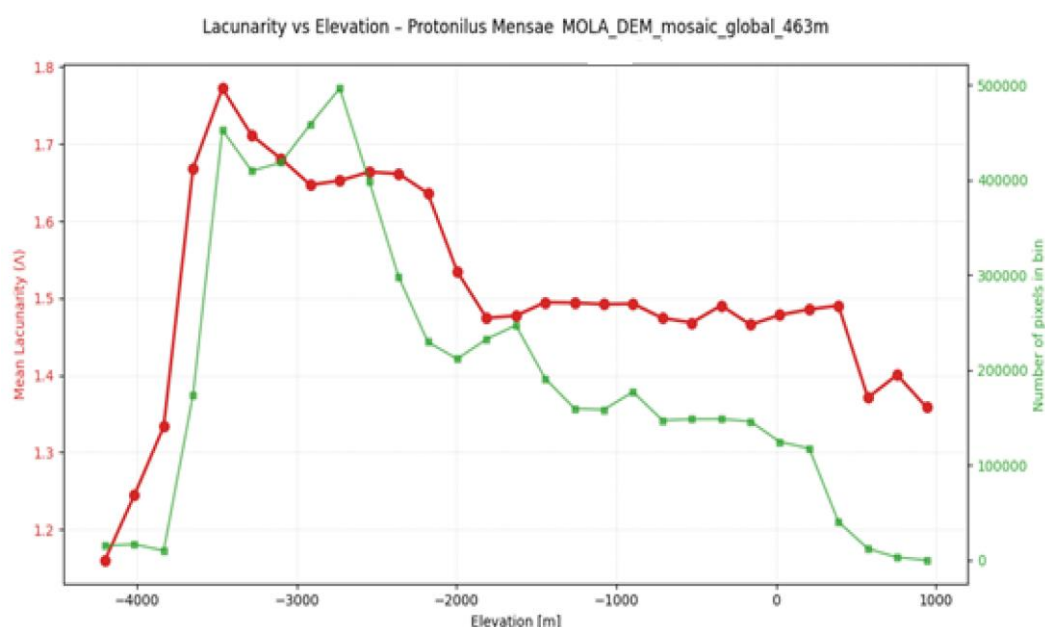


Fig. 3. Elevation Dependence of Surface Lacunarity in Protonilus Mensae region

The observed lacunarity–elevation relationship supports a model in which surface heterogeneity in Protonilus Mensae is maximized at intermediate elevations, corresponding to the fretted terrain. Lowest and highest elevations exhibit reduced lacunarity, representing smoother surfaces. The combined fractal and lacunarity patterns suggest that the present-day morphology likely represents a degraded state of an initially more rugged landscape. Later, it was subjected to intense asteroid bombardment, as

evidenced by numerous smaller and several large astroblemes (impact craters). This assumption is also supported by the Slope map of the studied area (Fig.4), where it is visible that the steepest (>10 degrees) are the rims of the craters and the slopes of the smaller plateaus.

These results are in accordance with later geomorphological evolution of the area driven by ice, sublimation (freeze-thaw cycle), and erosional processes, which modified initial topographic heterogeneity. Over the course of hundreds of

millions of years, erosion processes have shaped the terrain to its present appearance.

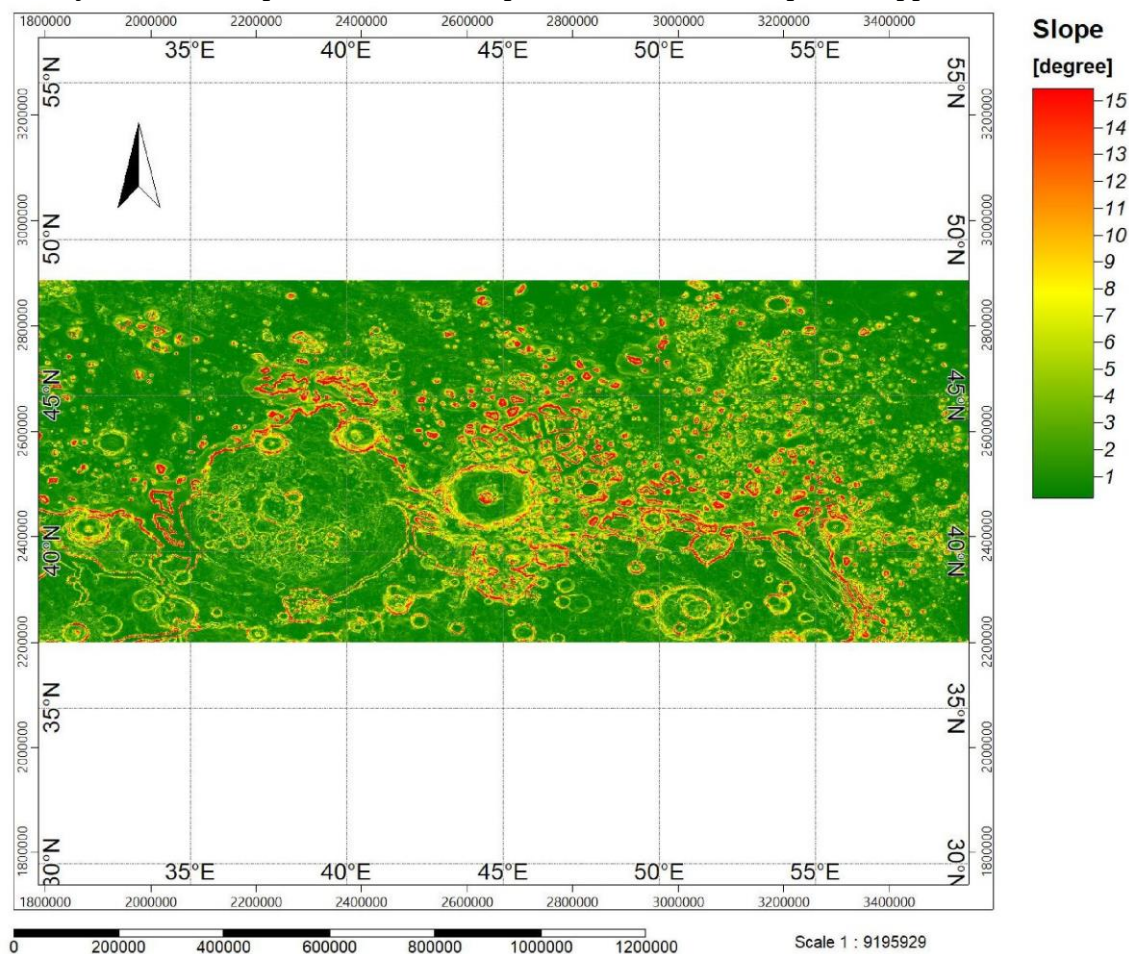


Fig. 4. Slope distribution map of Protonilus Mensae

CONCLUSION

This study presents a quantitative assessment of topographic complexity in Protonilus Mensae using variogram-based fractal dimension and lacunarity analyses. The results demonstrate that the region exhibits a fractal organization, with a mean fractal dimension of 2.352 and consistently high coefficients of determination (R^2 mean = 0.959), confirming well-defined power-law scaling of surface roughness.

The spatial distribution of fractal dimension values indicates that Protonilus Mensae is characterized predominantly by low to moderate surface complexity. Strongly rugged, highly multiscale terrain occupies only limited and spatially isolated areas, while most of the region displays intermediate fractal dimension values.

This pattern suggests low spatial anisotropy and a relatively uniform organization of surface roughness.

The lacunarity analysis further reveals a systematic relationship between surface heterogeneity and elevation. Surface heterogeneity is minimized at the lowest and highest hypsometric levels and maximized at intermediate elevations, coinciding with the core fretted terrain. This elevation-dependent behavior indicates that the spatial arrangement of mesas, troughs, and valleys is not random. The combined fractal and lacunarity analysis suggest that the present-day landscape represents a degraded state of an initially more rugged topography.

This study demonstrates that the combined use of fractal dimension and lacunarity approach provides a quantitative tool for characterizing

complex Martian terrains. The approach effectively captures both the scaling behavior and spatial organization of topography, offering different view about the geomorphological evolution of Mars. The study can be considered as innovative approach for ecology investigation and example of surface degradation of the Martian terrain in the region of Protonilus Mensae.

REFERENCES

- Albee, A. L., Arvidson, R. E., Palluconi, F., & Thorpe, T. (2001). Overview of the Mars Global Surveyor mission. *Journal of Geophysical Research: Planets*, 106(E10), 23291-23316. <https://doi.org/10.1029/2000JE001306>
- Allain, C., & Cloitre, M. (1991). Characterizing the lacunarity of random and deterministic fractal sets. *Physical Review A*, 44(6), 3552-3558. <https://doi.org/10.1103/PhysRevA.44.3552>
- Baker, M., et al. (2010). Flow patterns of lobate debris aprons and lineated valley fill north of Ismeniae Fossae, Mars: Evidence for extensive mid-latitude glaciation in the Late Amazonian. *Icarus*, 207, 186-209.
- Ching, D., Taylor, G. J., & Mouginis-Mark, P. J. (1993). Fractal dimensions of rampart impact craters on Mars. *Lunar and Planetary Science Conference*, 24, 283-284.
- Conrad, O., Bechtel, B., Bock, M., Dietrich, H., Fischer, E., Gerlitz, L., Wehberg, J., Wichmann, V., & Böhner, J. (2015). System for Automated Geoscientific Analyses (SAGA) v. 2.1.4. *Geoscientific Model Development*, 8, 1991-2007. <https://doi.org/10.5194/gmd-8-1991-2015>
- Deliege, A., Kleyntssens, T., & Nicolay, S. (2016). The fractal nature of Mars topography analyzed via the wavelet leaders method. In S. Latifi (Ed.), *Information technology: New generations* (Vol. 448). Springer. https://doi.org/10.1007/978-3-319-32467-8_117
- Demin, S. A., Andreev, A. O., Demina, N. Y., & Nefediev, Y. A. (2017). The fractal analysis of the gravitational field and topography of Mars. *Journal of Physics: Conference Series*, 929, 012002. <https://doi.org/10.1088/1742-6596/929/1/012002>
- Gallant, J. C., Moore, I. D., Hutchinson, M. F., et al. (1994). Estimating fractal dimension of profiles: A comparison of methods. *Mathematical Geology*, 26, 455-481.
- Iliev, R., & Rangelov, B. (2020). Fractal analysis of the mesas within Deuteronilus Mensae region of Mars. *TO Physics Journal*, 5, 68-71.
- Iliev, R., & Rangelov, B. (2025). Fractal nature of Martian terrain. *Aerospace Research in Bulgaria*, 37, 183-191. <https://doi.org/10.3897/arb.v37.e18>
- Landais, F., Schmidt, F., & Lovejoy, S. (2015). Universal multifractal Martian topography. *Nonlinear Processes in Geophysics*, 22, 713-722. <https://doi.org/10.5194/npg-22-713-2015>
- Madeleine, J., Forget, F., Head, J., Levrard, B., & Montmessin, F. (2007). Exploring the northern mid-latitude glaciation with a general circulation model. *Seventh International Conference on Mars (Abstract 3096)*.
- Mandelbrot, B. B. (1983). *The fractal geometry of nature*. W. H. Freeman.
- Mark, D. M., & Aronson, P. B. (1984). Scale-dependent fractal dimensions of topographic surfaces: An empirical investigation with applications in geomorphology and computer mapping. *Mathematical Geology*, 16(7), 671-683.
- Morgan, G. A., Head, J. W., & Marchant, D. R. (2009). Lineated valley fill (LVF) and lobate debris aprons (LDA) in the Deuteronilus Mensae northern dichotomy boundary region, Mars: Constraints on the extent, age, and episodicity of Amazonian glacial events. *Icarus*, 202(1), 22-38. <https://doi.org/10.1016/j.icarus.2009.02.017>
- Pardo-Iguzquiza, E., & Dowd, P. (2022). Fractal analysis of the Martian landscape: A study of kilometre-scale topographic roughness. *Icarus*, 372, 114727.
- Plotnick, R. E., Gardner, R. H., & O'Neill, R. V. (1993). Lacunarity indices as measures of landscape texture. *Landscape Ecology*, 8(3), 201-211. <https://doi.org/10.1007/BF00125351>
- Sharp, R. P. (1973). Mars: Fretted and chaotic terrains. *Journal of Geophysical Research*, 78, 4073-4083. <https://doi.org/10.1029/JB078i020p04073>
- Smith, D. E., Zuber, M. T., Frey, H. V., Garvin, J. B., Head, J. W., Muhleman, D. O., Pettengill, G. H., et al. (2001). *Mars Orbiter*

Laser Altimeter experiment summary after the first year of global mapping of Mars. *Journal of Geophysical Research: Planets*, 106(E10), 23689-23722. <https://doi.org/10.1029/2000JE001364>

Soare, R. J., Williams, J.-P., Hepburn, A. J., & Butcher, F. E. G. (2022). A billion or more years of possible periglacial/glacial cycling in Protonilus Mensae, Mars. *Icarus*, 385, 115115. <https://doi.org/10.1016/j.icarus.2022.115115>

Stepinski, T. F., Marinova, M. M., McGovern, P. J., & Clifford, S. M. (2002).

Fractal analysis of drainage basins on Mars. *Geophysical Research Letters*, 29(8). <https://doi.org/10.1029/2002GL014666>

Turcotte, D. L. (1987). A fractal interpretation of topography and geoid spectra on the Earth, Moon, Venus, and Mars. *Journal of Geophysical Research*, 92, 597-601.

Voss, R. F. (1986). Characterization and measurement of random fractals. *Physica Scripta*, 13, 27. <https://doi.org/10.1088/0031-8949/1986/T13/004>.

Rosen Iliev, Boyko Rangelov

Space Research and Technology Institute,
Bulgarian Academy of Sciences
Sofia 1113, Bulgaria

E-mails: ilievrosen@space.bas.bg,
brangelov@gmail.com