

NONLINEAR ADAPTIVE FAULT-TOLERANT CONTROL FOR LPV TIME-DELAY PEM FUEL CELL SYSTEMS

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Abstract: This paper addresses the design of an active Fault-Tolerant Control (FTC) strategy for Polymer Electrolyte Membrane Fuel Cell (PEMFC) systems exhibiting time-delay characteristics. An adaptive observer-based approach is proposed using a Linear Parameter Varying (LPV) model and Unknown Input Observer (UIO) techniques to estimate actuator faults and generate residuals. The proposed method leverages Linear Matrix Inequality (LMI) conditions to ensure the stability and convergence of the observer in the presence of parameter variations and delays. The fault estimation and control law are integrated to compensate for faults such as compressor mass flow and return manifold orifice deviations. Simulation results validate the robustness and efficiency of the control strategy under various fault scenarios.

Keywords: Fault tolerant control, Fuel cell systems, Time delay systems, LPV, UIO

1. INTRODUCTION

Fuel cells are increasingly recognized as promising alternative power sources due to their high efficiency, low emissions, and suitability for a wide range of applications, including automotive, portable, and stationary power systems. Among them, the Polymer Electrolyte Membrane Fuel Cell (PEMFC) stands out for its low operating temperature, fast start-up capability, and high power density. However, accurate modeling and control of PEMFC systems remain challenging because of their nonlinear dynamics, parameter variations, and the presence of time delays in the air supply and fuel delivery subsystems.

In particular, the air supply system of a PEMFC, which includes the compressor, supply manifold, and return manifold, introduces complex dynamics and delays that can degrade system performance and compromise fault diagnosis capabilities. To ensure reliable operation and extend the system's lifespan, advanced Fault Detection and Isolation (FDI), Fault Diagnosis (FDD), and Fault-Tolerant Control (FTC) strategies are essential. These approaches rely heavily on observer-based techniques to estimate internal states and detect potential actuator faults as in model-based control [1, 2], model-based diagno-

sis [3, 4], and FTC design [5]. Pukrushpan et al.[6] proposed a comprehensive model incorporating auxiliary components such as the air supply system, compressor, return manifold, and humidifier. This model was later used with an LQR (Linear Quadratic Regulator) strategy to improve system performance.

In recent years, considerable research has focused on estimation and diagnosis in fuel cell systems. Several modern estimators have been proposed, including neural network-based approaches and adaptive filtering methods such as adaptive unscented Kalman filters (UKF), often designed using data-driven or LPV-based modeling frameworks [3, 7]. Fuchs et al. [8] present a methodology to design a nonlinear distributed parameter observer to estimate internal temperature profiles in PEMFCs. Du et al. [9] introduced a fuel cell concentration observer design workflow, covering model-based design of experiments, their execution, nonlinear identification, and observer design with experimental validation.

The work in [10] proposed a nonlinear observer based on second-order sliding mode techniques to estimate the hydrogen partial pressure using measurements of stack voltage, current, and anode pressures. To address PEMFC durability and cost



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Table 1: Nomenclature

Variable	Description	Value
$k_{rm,out}$	Return manifold outlet orifice	0.3629×10^{-5} (kg/s/Pa)
$k_{ca,out}$	Cathode outlet orifice constant	0.2177×10^{-5}
$k_{sm,out}$	Manifold outlet orifice constant	0.3629×10^{-5}
η_{cp}	Compressor efficiency	0.8
γ	Ratio of specific heats of air	1.4
R_a	Air gas constant	286.9 J/(kg · K)
R_{O_2}	Oxygen gas constant	256.9 J/(kg · K)
R_{H_2}	Hydrogen gas constant	4124.3 J/(kg · K)
M_{H_2}	Hydrogen molar mass	2.016×10^{-3} kg/mol
M_{O_2}	Oxygen molar mass	32×10^{-3} kg/mol
k_t	Compressor motor torque constant	0.0153 N · m/A
k_v	Compressor motor voltage constant	0.0153 V/(rad/s)
K_1	Proportional gain	2.1
K_2	Nominal pressure drop coefficient	0.94
C_p	Specific heat capacity of air	1004 J/(mol · K)
T_{atm}	Atmospheric temperature	298.15 K
T_{sm}	Supply manifold temperature	300 K
T_{rm}	Return manifold temperature	300 K
T_{st}	Stack temperature	350 K
V_{ca}	Cathode volume	0.01 m ³
V_{rm}	Return manifold volume	0.005 m ³
V_{an}	Anode volume	0.005 m ³
n	Number of cells in stack	381
F	Faraday constant	96485 C/mol
W_{cp}	Compressor outlet mass flow rate	0.1 kg/s
p_{atm}	Atmospheric pressure	101325 Pa

challenges in automotive applications, Piffard et al. [11] proposed a sliding mode observer to estimate nitrogen build-up on the anode side. However, their method lacked sufficient precision for FDD applications, only providing average values of nitrogen and humidity.

Recent advancements have focused on observer and control design for nonlinear time-delay systems using fuzzy modeling frameworks. For instance, Zhang et al. [12] introduced a delay-dependent observer design for Takagi–Sugeno (T–S) fuzzy systems with unknown inputs, ensuring robust performance via Linear Matrix Inequalities (LMIs). Their approach improves the accuracy of state estimation under bounded delays. In parallel, Chen et al. [13] proposed a refined TDC scheme for PEMFCs that dynamically adapts to oxygen flow delays, enhancing system responsive-

ness under varying load conditions. Additionally, Liu and Zhao [14] developed a hybrid fuzzy-TDC structure for regulating hydrogen and oxygen stoichiometry in PEM fuel cells, effectively accounting for transport and measurement delays. Despite these efforts, further work is still needed to comprehensively address the cumulative effects of multiple time delays across subsystems.

Time delays are inherent in many engineering systems, including chemical processes, mechanical actuators, power electronics, and networked control architectures, primarily due to communication latency, transport phenomena, and sensor-actuator synchronization. These delays can significantly impact system stability and degrade control performance. Recent advances in delayed system analysis have focused on observer design and compensation techniques, both

in delay-independent and delay-dependent frameworks. Delay-dependent methods are generally less conservative and more effective in practice [15]. Readers interested in estimation methods for delayed systems can refer to recent overviews and theoretical developments in nonlinear observer design for time-delay systems [16, 17].

Recently, significant advances have been made in fault-tolerant control (FTC) strategies for PEMFC systems, particularly in addressing dynamic disturbances and actuator faults. A comprehensive and up-to-date review of FTC techniques for hydrogen-based energy systems was provided by Mezghani et al. [18], highlighting modern trends in robust, adaptive, and intelligent control architectures. In a more application-oriented study, Singh et al. [19] proposed a real-time FTC scheme for DC-DC converters in fuel cell electric vehicles (FCEVs), combining model predictive control with sensor fault estimation. Similarly, El-Zoghby et al. [20] developed a T-S fuzzy logic-based virtual actuator framework with delay compensation, enhancing fault accommodation in PEMFC-powered systems. While these approaches aim to improve transient performance and system resilience, the challenge of managing time delays in state estimation and fault compensation remains open. This paper contributes by addressing the combined problem of fault detection, isolation, and compensation (FDI/FTC) in PEMFC systems modeled with LPV structures and incorporating oxygen mass flow measurements with varying delay effects.

This paper is structured as follows: The Fuel Cell system description and modeling objective are stated in Section 2. The Section 3 describes the Observer stability and convergence in the presence of time delays are demonstrated in this section. Next, the simulation results of PEM fuel cell system with the compressor outlet mass flow rate and the return manifold outlet orifice actuators are tested to verify the performance are given in Section 4. Finally, the conclusion of this paper is elaborated in last section.

2. PROBLEM FORMULATION AND MODELING

2.1. PEMFC System Description

PEMFC supplies electricity through the electrochemical reaction from oxygen and hydrogen. It contains an electrolyte, a cathode and an anode. IA fuel cell operates by supplying hydrogen and oxygen to the anode and cathode, respectively, typically provided by an air compressor. PEMFC systems are the best choice for both portable and stationary applications due to their low emissions, low operating temperature, lightweight structure, high efficiency, and rapid start-up. The PEMFC plant whose structure is illustrated in Fig. 1 mainly includes : air compressor, hydrogen tank, a supply manifold and a return manifold.

2.2. PEM Fuel Cell Model

Regarding the modeling and estimation of time delays systems for fuel cells, very little work has been reported in the literature. Whereas a measurement of the cathode exhaust air (ODA) gas mass flow and oxygen content introduced a significant time delay. Then, this work incorporates this delay in the modeling of the PEMFC. Using the model presented by Pukrushpan et al.[6] considering the fact that a time delay related to the mass flow oxygen in the anode of the PEMFC can exist, leading to the dynamic model given by equation (1). This model represents a good behavior of a fuel cell stack and the observer and control community generally accept it.

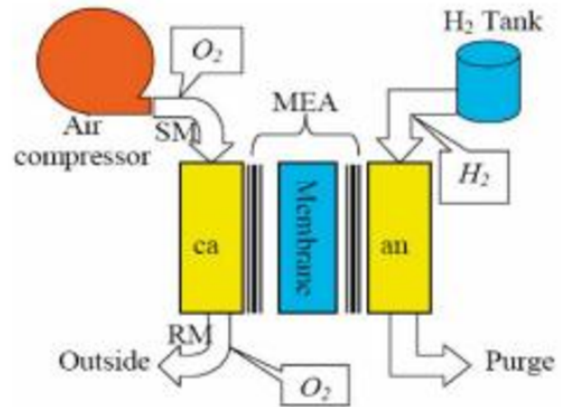


Fig. 1: SPeMFC system with auxiliary components

$$\begin{cases}
\dot{p}_{sm}(t) = \frac{\gamma R_a}{V_{sm} T_{atm} W_{cp1}} + \frac{1}{\eta_{cp}} \cdot \frac{p_{sm}(t)}{p_{atm}} \cdot \frac{1}{\lambda - 1} \cdot \gamma \\
\quad + k_{sm,out} p_{sm}(t) - \frac{R_{O_2} T_{st}}{V_{ca} T_{sm}} m_{O_2,ca}(t - \tau) \\
\dot{p}_{rm}(t) = \frac{R_a T_{rm}}{V_{rm}} k_{ca,out} \cdot \frac{R_{O_2} T_{st}}{V_{ca}} m_{O_2,ca}(t - \tau) \\
\quad - p_{rm}(t) - k_{rm,out} (p_{rm}(t) - p_{atm}(t)) \\
\dot{m}_{O_2,ca}(t) = k_{sm,out} p_{sm}(t) - k_{sm,out} \\
\quad + k_{ca,out} \frac{R_{O_2} T_{st}}{V_{ca}} m_{O_2,ca}(t) + k_{ca,out} p_{rm}(t) \\
\dot{m}_{H_2,an}(t) = K_1 K_2 p_{sm}(t) - \frac{R_{H_2} T_{st}}{V_{an}} m_{H_2,an}(t)
\end{cases} \quad (1)$$

$$x(t) = \begin{bmatrix} p_{sm}(t) \\ p_{rm}(t) \\ m_{O_2,ca}(t) \\ m_{H_2,an}(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} p_{sm}(t) \\ p_{rm}(t) \\ m_{H_2,an}(t) \end{bmatrix}$$

$$u(t) = \begin{bmatrix} W_{cp}(t) \\ k_{rm,out}(t) \end{bmatrix}$$

Consequently, the system (1) can be realized as follows:

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t - \tau) + B_i u(t) \\ y(t) = C x(t) \end{cases} \quad (2)$$

$$A_0 = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & a_{42} & 0 & a_{44} \end{bmatrix}$$

$$a_{11} = -\frac{\gamma R_a k_{sm,out} T_{sm}}{V_{sm}}$$

$$a_{22} = -\frac{R_a k_{ca,out} T_{rm}}{V_{rm}}$$

$$a_{31} = K_1 K_2, \quad a_{33} = -\frac{K_1 R_{H_2} T_{st}}{V_{an}}$$

$$a_{41} = k_{sm,out}, \quad a_{42} = k_{ca,out}, \quad a_{44} = \chi_{44}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & \chi_{14} \\ 0 & 0 & 0 & \chi_{24} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_1(p_{sm}) & 0 \\ 0 & b_2(p_{sm}) \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\chi_{14} = \frac{\gamma R_a k_{sm,out} R_{O_2}}{T_{st} T_{sm} V_{ca} V_{sm}},$$

$$\chi_{24} = \frac{R_a k_{ca,out} R_{O_2}}{T_{st} T_{sm} V_{ca} V_{sm}},$$

$$\chi_{44} = -\frac{(k_{sm,out} + k_{ca,out}) R_{O_2} T_{st}}{V_{ca}},$$

$$b_1(p_{sm}) = \frac{\gamma R_a T_{atm} (\eta_{cp} + 1)}{V_{sm} \eta_{cp}},$$

$$b_2(p_{sm}) = -\frac{R_a T_{rm}}{V_{rm}} (p_{rm} - p_{atm})$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^c$ is the control input, and $y(t) \in \mathbb{R}^m$ is the output vector. Matrices A_0 , A_1 , and C have appropriate dimensions, and B_i for $i = 1, \dots, r$ are matrices dependent on the time-varying scheduling parameters ρ_i , such that:

$$B(\rho) = \sum_{i=1}^r \rho_i B_i$$

The scheduling variables ρ_i are the LPV subsystem weights satisfying:

$$\sum_{i=1}^r \rho_i = 1, \quad 0 \leq \rho_i \leq 1$$

In this case, we use the following singular delayed LPV model to represent the considered PEMFC system under actuator faults:

$$\begin{cases} E\dot{x}(t) = \sum_{i=0}^n \rho_i A_i x(t) + \sum_{i=0}^n \rho_i A_{\tau i} x(t - \tau) \\ \quad + \sum_{i=0}^n \rho_i B_i u(t) + F f_a(t) \\ y(t) = C x(t) \\ x(t) = \varphi(t) \end{cases} \quad (3)$$

Assumptions:

- The pair (A_i, C) is observable for all $i = 0, \dots, n$.
- The matrix C is of full row rank.

Lemma 1. Consider two matrices P and Z , where P is a symmetric positive definite matrix and Z is full column rank. Then the following inequality holds:

$$ZPZ^T > 0$$

3. ACTIVE FAULT TOLERANT CONTROL DESIGN

In this section, we design an unknown input observer (UIO) for the delayed PEMFC system with LPV representation. The convergence of this observer will be analyzed. On Fig.2 a control scheme for FTC is presented, which is designed to maintain performance close to nominal operation despite actuator faults.

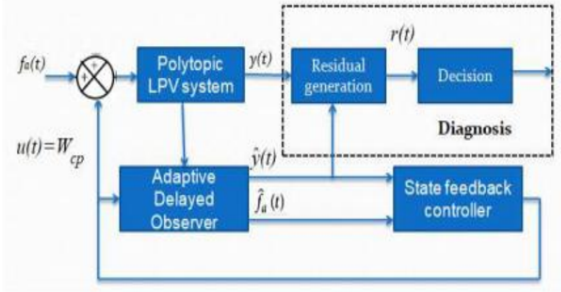


Fig. 2: Fault Tolerant Control Scheme

3.1. Design of UIO for Delayed PEMFC System

We propose an Unknown Input Observer (UIO) design for delayed PEMFC systems based on LPV systems modeling. The existence and convergence conditions are formulated as LMIs terms. In this approach, the adaptive observer to unknown inputs is formulated as follows:

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^n \rho_i (N_i z(t) + N_{\tau i} z(t-\tau) + L_{1i} (y(t) - \hat{y}(t)) \\ &\quad + L_{2i} (y(t-\tau) - \hat{y}(t-\tau)) + G_i u(t) + T \hat{f}_a(t)) \\ \hat{x}(t) &= z(t) + H_2 y(t) \\ \hat{y}(t) &= C \hat{x}(t) \\ r(t) &= y(t) - \hat{y}(t) \\ r(t-\tau) &= y(t-\tau) - \hat{y}(t-\tau) \\ \hat{f}_a(t) &= \Gamma S \left(r(t) + \sigma \int r(t) dt \right) \end{aligned} \quad (4)$$

where $z(t) \in \mathbb{R}^n$ and $\hat{x}(t) \in \mathbb{R}^n$ denote the observer state and the estimated state, respectively;

$\hat{y}(t) \in \mathbb{R}^p$ is the estimated output. The matrices $N_i, N_{\tau i}, L_{1i}, L_{2i}, G_i, T$, and H_2 are observer gains of appropriate dimensions.

3.2. FTC for an Actuator Fault

In this part, an active FTC approach for actuator faults is elaborated. The control law compensating for actuator faults is defined as:

$$u(t) = -K \hat{x}(t) + \sum_{i=1}^r \rho_i K_{f_i} \hat{f}_a(t) \quad (5)$$

where K is the state feedback gain, and $K_{f_i} = -B_i^+ F$.

Let $H_1 \in \mathbb{R}^{n \times n}$ such that:

$$H_1 E = I_n - H_2 \bar{C} \quad (6)$$

Estimation errors are defined as:

$$e_x(t) = x(t) - \hat{x}(t) \quad (7)$$

$$e_f(t) = f_a(t) - \hat{f}_a(t) \quad (8)$$

Then,

$$e_x(t) = H_1 E x(t) - z(t) \quad (9)$$

and the error dynamics is given as:

$$\begin{aligned} \dot{e}_x(t) &= \sum_{i=1}^n \rho_i \left(N_i e_x(t) + N_{\tau i} e_x(t-\tau) \right. \\ &\quad + (H_1 A_i - L_{1i} C - N_i H_1 E) x(t) \\ &\quad + (H_1 A_{\tau i} - L_{2i} C - N_{\tau i} H_1 E) x(t-\tau) \\ &\quad \left. + (H_1 F - T) e_f(t) + (H_1 B_i - G_i) u(t) \right) \end{aligned} \quad (10)$$

If the following conditions hold:

$$H_1 E + H_2 C = I_n \quad (11)$$

$$H_1 A_i - N_i H_1 E = L_{1i} C \quad (12)$$

$$H_1 A_{\tau i} - N_{\tau i} H_1 E = L_{2i} C \quad (13)$$

$$T = H_1 F_i \quad (14)$$

$$G_i = H_1 B \quad (15)$$

then the dynamic error equation becomes:

$$\begin{cases} \dot{e}_x(t) = \sum_{i=1}^n \rho_i (N_i e_x(t) + N_{\tau i} e_x(t-\tau)) \\ e_y(t) = C e_x(t) \end{cases} \quad (16)$$

$$\dot{e}_f(t) = \dot{f}_a(t) - \dot{\hat{f}}_a(t) \quad (17)$$

$$\dot{e}_f(t) = \dot{f}_a(t) - \Gamma S(\dot{e}_y(t) + \sigma e_y(t)) \quad (18)$$

Lemma 2

The delayed PEMFC system with LPV representation is stable if there exist positive definite symmetric matrices P and Q_j for $j = 1..n$ satisfying:

$$\begin{bmatrix} b_1 & b_2 & \cdots & b_{n+1} \\ * & b_{n+2} & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & b_{2n+1} \end{bmatrix} < 0 \quad (19)$$

where:

- $b_1 = PN_0 + N_0^T P + Q_1 + \cdots + Q_n$
- $b_2 = PN_1, \dots, b_{n+1} = PN_n$
- $b_{n+2} = -(1 - \alpha_1)Q_1, \dots, b_{2n+1} = -(1 - \alpha_n)Q_n$

To demonstrate the stability of the considered delayed PEMFC system, the following Lyapunov-Krasovskii function is defined:

$$V(e(t)) = e^T(t)Pe(t) + \sum_{j=1}^n \int_{t-\tau_j(t)}^t e^T(s)Q_j e(s) ds \quad (20)$$

where P and Q_j are positive definite symmetric matrices for $j = 1..n$.

The time derivative of $V(e(t))$ is:

$$\begin{aligned} \dot{V}(e(t)) &= e^T(t)PN_0 e(t) + e^T(t)PN_0^T e(t) \\ &+ \sum_{j=1}^n \left(e^T(t)PN_j e(t - \tau_j(t)) \right. \\ &+ e^T(t - \tau_j(t))PN_j^T e(t) + e^T(t)Q_j e(t) \\ &\left. - (1 - \dot{\tau}_j(t))e^T(t - \tau_j(t))Q_j e(t - \tau_j(t)) \right) \end{aligned}$$

Seeing that $\dot{\tau}_j(t) \leq \alpha_j$ for $j = 1..n$, we get:

$$\dot{V}(e(t)) \leq \begin{bmatrix} e(t) \\ e(t - \tau_1) \\ e(t - \tau_2) \\ \vdots \\ e(t - \tau_n) \end{bmatrix}^T \Xi \begin{bmatrix} e(t) \\ e(t - \tau_1) \\ e(t - \tau_2) \\ \vdots \\ e(t - \tau_n) \end{bmatrix} \quad (21)$$

Under the stability theory of Lyapunov, the obtained error can be stable if $\dot{V}(e(t)) < 0$. This approach then consists in determining the nominal and additive control of the LPV delay system, which will be done using the LMI approach. As a result, the estimate of the actuator fault amplitude will be made by the adaptive observer with unknown inputs [5].

3.3. Proceedings of Calculating the UIO

This subsection outlines the procedure for computing the unknown input observer (UIO) gains.

$$\begin{bmatrix} H_1 & H_2 \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix} = I_n \quad (22)$$

Let the matrix $H = \begin{bmatrix} H_1 & H_2 \end{bmatrix}$ be unknown, and define $Y = \begin{bmatrix} E \\ C \end{bmatrix}$. If

$$\text{rank}(Y) = n \quad (23)$$

Then the solution to equation (29) is given by:

$$H = I_n Y^+ \quad (24)$$

where Y^+ is the pseudo-inverse of Y of dimension $n \times (n + m)$, computed as:

$$Y^+ = (Y^T Y)^{-1} Y^T \quad (25)$$

Assuming $Y_1 \in \mathbb{R}^{n \times n}$, $Y_2 \in \mathbb{R}^{n \times m}$ and defining $X = I_n - Y Y^+$, then equation (31) can be rewritten as:

$$H = \begin{bmatrix} H_1 & H_2 \end{bmatrix} = I_n \begin{bmatrix} Y_1^+ & Y_2^+ \end{bmatrix} + K \begin{bmatrix} X_1 & X_2 \end{bmatrix} \quad (26)$$

Introducing a variable K_i :

$$K_i = L_i - N_i H_2 \quad (27)$$

for $i = 0..n$. Thus,

$$N_i = H_1 A_i - K_i C \quad (28)$$

Substituting H_1 into (28), we obtain:

$$N_i = H_{10} A_i + K X_1 A_i - K_i C \quad (29)$$

with $H_{10} = I_n Y_1^+$.

3.4. Convergence and Stability Analysis

Theorem 1. If there exist symmetric, positive definite matrices P and Q_j for $j = 1..n$ and matrices M and P_j for $j = 0..n$ such that:

$$\begin{bmatrix} \Delta_0 & \Delta_1 & \Delta_2 & \cdots & \Delta_n \\ * & c_1 & 0 & \cdots & 0 \\ * & * & c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & c_n \end{bmatrix} < 0 \quad (30)$$

where:

$$c_i = -(1 - \alpha_i)Q_i, \quad \text{for } i = 1, 2, \dots, n$$

$$\begin{aligned} \Delta_0 &= PH_{10}A_0 + MX_1A_0 - P_0C \\ &\quad + (PH_{10}A_0 + MX_1A_0 - P_0C)^T \\ &\quad + Q_1 + \cdots + Q_n, \\ \Delta_j &= PH_{10}A_j + MX_1A_j - P_jC, \quad j = 1..n \end{aligned} \quad (31)$$

Then, the observer defined in Equation (4) exists and the estimation error converges to zero.

The observer gain matrix is given by:

$$K = P^{-1}M \quad (32)$$

$$K_j = P^{-1}P_j, \quad j = 1, \dots, n \quad (33)$$

Proof: Replacing equation (29) into Lemma 2 (equation (19)).

Algorithm 1. UIO Design

1. Compute $H_{10} = I_n Y_1^+$ and extract X_1 from the first n columns of X .
2. Solve the LMI in equation (30) to obtain the matrices P , M , Q_j for $j = 1..n$, and P_j for $j = 0..n$.
3. Calculate K and K_i for $i = 0..r$ from equations (32) and (33), respectively.
4. Compute G_i for $i = 1..r$, H_2 , and N_i from equations (15), (11), and (19), respectively.
5. Determine L_i for $i = 0..r$

4. SIMULATION RESULTS

We take the system (4) with the following matrices:

$$A = \begin{bmatrix} -21.8 & 0 & 0 & 0 \\ 0 & -37.4 & 0 & 0 \\ 1.97 & 0 & -6.0 \times 10^8 & 0 \\ 3.63 \times 10^{-6} & 2.17 \times 10^{-6} & 0 & -52.8 \end{bmatrix}$$

$$A_r = \begin{bmatrix} 0 & 0 & 0 & 1.99 \times 10^8 \\ 0 & 0 & 0 & 3.41 \times 10^8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 9.94 \times 10^5 & 0 \\ 0 & -4.94 \times 10^{11} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 9.94 \times 10^5 & 0 \\ 0 & -2.06 \times 10^{13} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 1.45 \times 10^7 & 0 \\ 0 & -4.94 \times 10^{11} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_4 = \begin{bmatrix} 1.45 \times 10^7 & 0 \\ 0 & -2.06 \times 10^{13} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The membership functions can be selected as:

$$\rho_1(p_{sm}, p_{rm}) = \frac{p_{sm_max} - p_{sm}}{p_{sm_max} - p_{sm_min}} \cdot \frac{p_{rm_max} - p_{rm}}{p_{rm_max} - p_{rm_min}}$$

$$\begin{aligned}\rho_2(p_{sm}, p_{rm}) &= \frac{p_{sm_max} - p_{sm}}{p_{sm_max} - p_{sm_min}} \cdot \frac{p_{rm} - p_{rm_min}}{p_{rm_max} - p_{rm_min}} \\ \rho_3(p_{sm}, p_{rm}) &= \frac{p_{sm} - p_{sm_min}}{p_{sm_max} - p_{sm_min}} \cdot \frac{p_{rm_max} - p_{rm}}{p_{rm_max} - p_{rm_min}} \\ \rho_4(p_{sm}, p_{rm}) &= \frac{p_{sm} - p_{sm_min}}{p_{sm_max} - p_{sm_min}} \cdot \frac{p_{rm} - p_{rm_min}}{p_{rm_max} - p_{rm_min}}\end{aligned}$$

Solving the LMIs according to the algorithm steps, the gains of the UIO can be computed as follows:

$$N_0 = 10^8 \times \begin{bmatrix} -0.930 & -0.545 & 0.000 & 0.994 \\ -1.5942 & -0.934 & 0.000 & 1.704 \\ -0.000 & 0.0000 & -0.0000 & 0.0000 \\ 0.000 & 0.000 & -0.000 & -0.000 \end{bmatrix}$$

$$L_0 = 10^8 \times \begin{bmatrix} 0.9301 & 0.0803 & -0.2727 \\ 1.5942 & 0.1376 & -0.4674 \\ 0.0000 & -0.0000 & -3.0314 \\ -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$$

$$G_1 = 10^{11} \times \begin{bmatrix} 0.0000 & 0 \\ 0 & -2.4681 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_2 = 10^{13} \times \begin{bmatrix} 0.0000 & 0 \\ 0 & -1.0317 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_3 = 10^{11} \times \begin{bmatrix} 0.0001 & 0 \\ 0 & -2.4681 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_4 = 10^{13} \times \begin{bmatrix} 0.0000 & 0 \\ 0 & -1.0317 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The fault signals of the actuator are given by the following equations:

$$\begin{aligned}f_1(t) &= \begin{cases} W_{cp}, & \text{si } 20 \leq t \leq 30 \\ 0, & \text{sinon} \end{cases} \\ f_2(t) &= \begin{cases} 10^{-3} \sin(0.5t), & \text{si } 40 \leq t \leq 50 \\ 0, & \text{sinon} \end{cases}\end{aligned}$$

For each case, we will represent the evolution of the outputs of the stack before and after compensation of the faults.

Some simulations results are shown on Figs. 3, 4, 5 and 6.

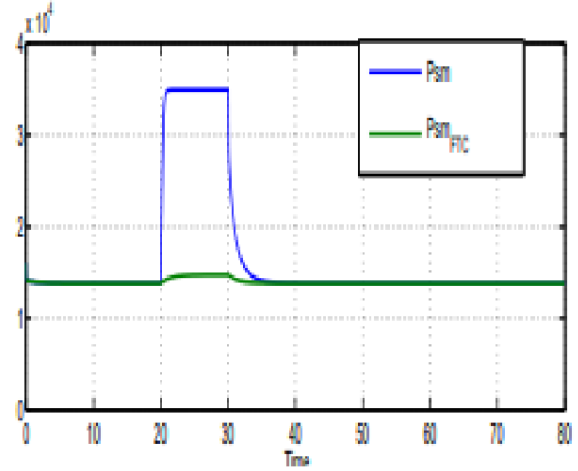


Fig. 3: Evolution of the supply manifold pressure with and without fault compensation.

Referring to the simulation results, it can be deduced that the control law developed based on the adaptive observer is able to compensate various type of actuator faults.

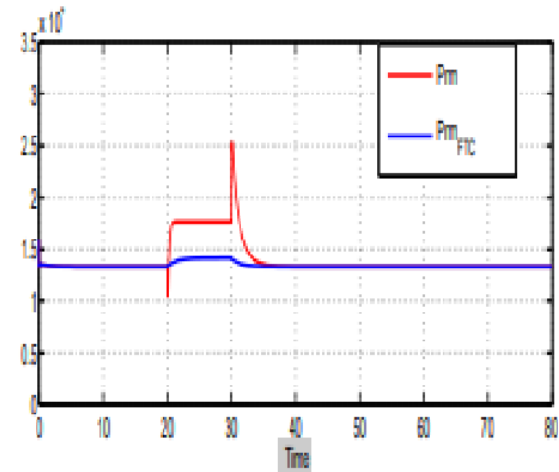


Fig. 4: Return manifold pressure before and after control

5. CONCLUSION

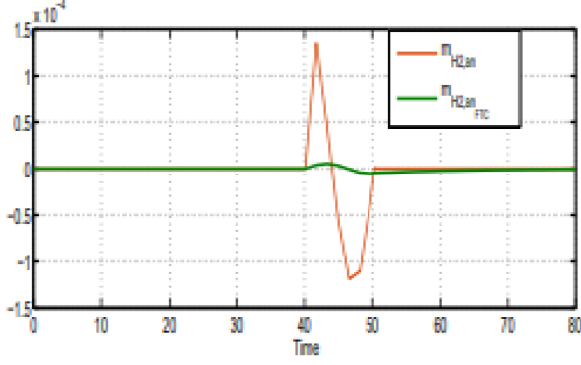


Fig. 5: Anode Hydrogen mass before and after control

This study presents a robust active fault-tolerant control strategy for PEMFC systems modeled via an LPV model with time-delay representation. An adaptive UIO scheme was developed to estimate actuator faults and a control scheme was proposed to ensure system stability and performance.

Simulation results confirm the controller's effectiveness in compensating for various actuator faults despite fault occurrences. Future work will focus on experimental validation and integration within hybrid energy systems under real-time constraints.

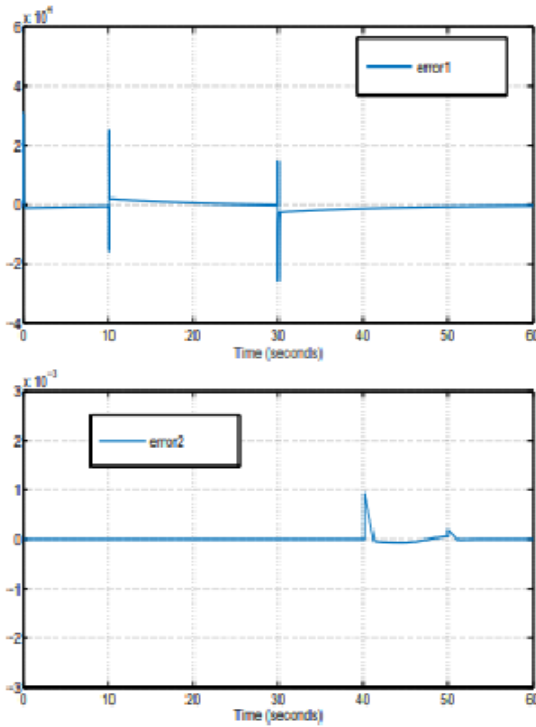


Fig. 6: Actuator fault estimation errors, r_1 and r_2 .

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